Student conceptual resources for understanding electric circuits

Lauren C. Bauman,[1] Brynna Hansen,[2] Lisa M. Goodhew,[2] and Amy D. Robertson[2]
[1]*Department of Physics, University of Washington, Box 351560, Seattle, Washington 98195, USA*
[2]*Department of Physics, Seattle Pacific University, Seattle, Washington 98119, USA*

(Received 7 February 2023; accepted 17 June 2024; published 11 October 2024)

Physics Education Research has a rich history of identifying common student ideas about specific physics topics. In the context of electric circuits, existing research on students' ideas has primarily focused on misconceptions, misunderstandings, and difficulties. In this paper, we take a resource-oriented approach to identifying common student ideas about circuits by characterizing ideas we see as generative "seeds of science" that could form the basis of more sophisticated understandings. Based on our analysis of 1557 university physics student responses to five conceptual questions, we identify four common resources for understanding circuits.

DOI: 10.1103/PhysRevPhysEducRes.20.020128

I. INTRODUCTION

Students' conceptual understanding of physics is "one of the earliest and most widely studied areas in physics education research" [1]. Starting more than 40 years ago, researchers began cataloging student ideas about physics, in most cases characterizing the difficulties or misconceptions evident in student reasoning about topics ranging from mechanics to circuits to waves and optics. This research had a significant impact on instruction in physics, from "rais[ing] consciousness among instructors about students' learning difficulties" [1]; "supply[ing]…technical knowledge about how students think and learn" in physics [2]; "provid[ing] a good source of ideas for problems, demonstrations, and laboratory experiments" [3]; to serving as a "sound base for informing the development of curriculum" [4]. Indeed, "knowledge of student ideas" is one component of the specialized knowledge for teaching that Shulman cites in his theory of pedagogical content knowledge [5]. Research on student ideas in physics has offered this knowledge to instructors and curriculum developers.

At the same time that researchers were beginning to document student misconceptions and difficulties in physics, a complementary theory of cognition was being developed to explain the variability and context sensitivity of student thinking [6–10]. This theory became known in physics education research as *resources theory*. In this theory, what we observe as in-the-moment student thinking is modeled as an amalgamation of pieces of knowledge that are being activated in context-sensitive ways. These pieces of knowledge are called *resources* and are theorized to be derived from students' experiences (including classroom experiences), to help make sense of the material world. Consistent with constructivism, resources are thought to be the building blocks for more formal understandings of physics [6,9,10].

While some work has been done to characterize specific resources that students bring to bear as they learn about physics topics (e.g., diSessa's force and motion p-prims [9] and Minstrell's facets [11,12]), much of the physics education research focusing on resources has been aimed at the development of theory that can explain and predict student thinking and learning. This research has been especially useful in drawing instructor attention to a variety of classroom phenomena and informing expectations of, for example, the context sensitivity of student thinking. Yet there are also opportunities for a resource framing to inform (and then complement) research like that which has been done to characterize students' difficulties and misconceptions about particular topics in physics; that is, there is new ground to be broken in identifying *common conceptual resources for understanding physics*. Such research has the potential to inform instructors' pedagogical content knowledge in ways similar to those named above: supplying knowledge about how students think and learn in physics, raising consciousness about student resources for learning physics, providing a source of knowledge for instructional tools and strategies, and informing curriculum development. Indeed, this kind of research was called for in 2000 [6]:

> […] Whereas the physics education research community has devoted substantial attention to studying the nature of student difficulties, it has paid little attention to documenting and systematizing extant ideas about student resources. Without that

attention, this knowledge remains mostly tacit and unexamined. I am arguing that it should become a primary agenda of the physics education research community to develop explicit accounts of student resources, to allow their exchange, review, and refinement.

Our work takes up this call, identifying common conceptual resources for understanding circuits, one of the most well-documented topics in physics when it comes to research on student thinking. Researchers have documented student ideas across grade levels, instructional settings, countries, and topics within circuits [13–22]. As with many topics in physics, the majority of this research focuses on students' misunderstandings, misconceptions, or difficulties. Our paper supplements this research, identifying some *common student resources for understanding electric circuits*—resources that may be "disciplinary progenitors" [23], "seeds of science" [24–26], or ideas that could be framed as generative for future learning (including those ideas developed in previous instruction). Whereas a number of studies have already identified common patterns in students' *incorrect* thinking (that instruction can address and replace), the research we present here provides instructors with knowledge of student ideas about circuits that curriculum and instruction can elicit, build from, and refine toward sophisticated scientific models.

In this paper, we start by reviewing themes from the existing literature on student ideas about circuits (Sec. II), then share our theoretical framework (Sec. III), summarize our methods (Sec. IV), and describe the common resources for understanding circuits we identified in student responses in our study (Sec. V). We close with a discussion and implications (Sec. VI).

II. STUDENT IDEAS ABOUT CIRCUITS: THEMES FROM EXISTING LITERATURE

Over the last four decades, researchers have extensively characterized students' ideas about circuits and have used insights from this research to develop concept inventories, design curricular materials, and prepare physics instructors. One of the most frequently documented findings in this literature is students' difficulty in conceptualizing and applying the concept of current. An often-cited example of this difficulty is the notion that the battery is a constant source of current, where students do not appear to recognize the role of resistance in determining the current through a circuit [13–15,17,18,21,27–30]. For instance, McDermott and Shaffer [18] write,

> Perhaps the most pervasive and persistent difficulty that students have with dc circuits is the belief that the battery is a constant source of current (i.e., the current through a battery always has the same value). They [students] often overlook the critical role played by resistance in determining the current.

Similarly, Cohen *et al.* [13] define the "constant current" misconception as the belief that "the current provided by a battery does not change when the external circuit is modified." They give the following example: When asked to answer the question shown in Fig. 1, a high school student from Cohen *et al.*'s study responded, "(a) is correct. When [element] N is removed, what happens is that all the current which was previously flowing in the main branch now flows through [element] M, and therefore M lights more strongly."

Another cluster of common difficulties reported in the literature focuses on students reasoning "sequentially," that is, reasoning as though the direction of current and order of elements matter for what happens in the circuit (e.g., whether or not the bulbs light, how bright the bulbs are, or whether there are changes in the current or voltage in the circuit) [18–21,27,30–34]. For example, Shipstone [21,33,34] characterizes sequential reasoning in terms of a model in which "current, as it progresses around the circuit, is influenced by each element that it encounters in turn" [33], rather than by the arrangement of the circuit as a

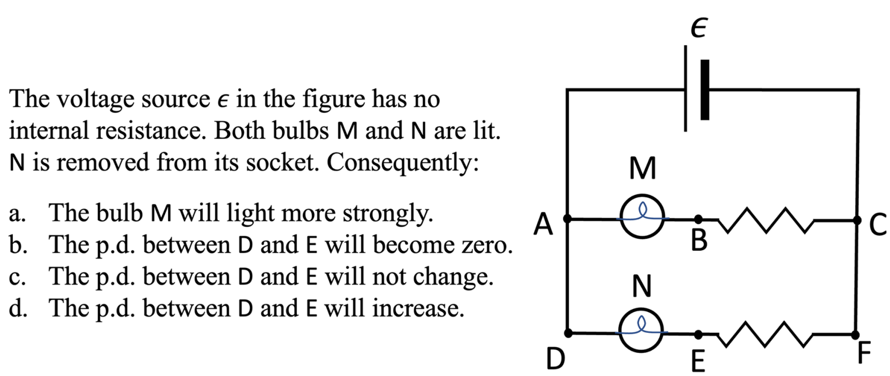


FIG. 1. Conceptual circuits question reproduced from R. Cohen, B. Eylon, and U. Ganiel, Am. J. Phys. **51**, 407 (1983), with the permission of AIP Publishing.

whole. That is, information about a change in the circuit is only "transmitted" as the current encounters each element, in the direction that current flows. In Shipstone's study, students were shown a simple circuit consisting of a battery, a bulb, and two resistors, R_1 and R_2. The battery, resistors and bulb were all in series, and the single bulb was placed in between R_1 and R_2. Students were asked how the values of two resistors, R_1 and R_2, would affect the brightness of a bulb placed between them. Students using sequential reasoning answered that the resistor placed "before" the bulb is the only resistor that would affect the bulb's brightness [34].

Relatedly, several studies report that students tend to reason locally rather than globally about circuit variables [13,21,22,32]. "Local reasoning" is characterized by a difficulty in recognizing that a change in one part of a circuit affects global variables. For example, a student using local reasoning may not realize that adding a resistor in parallel will lower the equivalent resistance of the circuit and in turn increase the current. Local reasoning is similar to sequential reasoning, in that students focus their attention on what is happening at a single point in the circuit, rather than treating the circuit as a system where all components are interconnected [22]. However, local reasoning is not the *same* as sequential reasoning, because students may reason locally without focusing on the order in which elements are "encountered" by the current.

The existing corpus of research, on the whole, communicates that students often struggle in learning about circuits and offers instructors tools and curriculum to diagnose and address difficulties [19–21,28,32–34]. Our research complements the existing literature by identifying and naming some of the common conceptual resources—i.e., patterns of potentially fruitful thinking—that students use to reason about electric circuits, encouraging instructors to build from student thinking, in addition to addressing or replacing student difficulties. We give evidence that students use these resources in response to a variety of conceptual questions and across several instructional contexts and discuss some of the ways that instructors may leverage these resources for students' learning.

III. THEORETICAL AND INSTRUCTIONAL MOTIVATIONS

Our research—and thus our methods—is motivated by the aim of producing insights that support instructors in noticing and responding to students' conceptual resources, an aim that is consistent with the orientation advanced by the resources theory of knowledge. We describe resources theory and further elaborate on our instructional motivations in this section.

A. Resources theory of knowledge

In resources theory, a *resource* is a piece of knowledge that is activated in real time and in context-sensitive ways [6–10,35–38]. Researchers have theorized extensively about the development, structure, and role of resources and have used resources theory to highlight the dynamic, emergent, complex-systems-like nature of student thinking. Our work draws extensively on the following tenets from resources theory:

1. *Resources are fundamentally sensible and generative for learning formal physics,* having been derived from a person's experience (including prior learning and classroom experiences), and then used to make sense of the material world [6,8–10,35,37,38]. For example, diSessa [9] says that phenomenological primitives ("p-prims," which we consider to be a kind of resource) such as "closer means stronger" are best understood as "serv[ing] individuals well in dealing effectively with the physical world," e.g., in making sense of it, interacting with it, etc. Smith *et al.* [10] define resources as "any feature of the learner's present cognitive state that can serve as a significant input to the process of conceptual growth," emphasizing the continuity between students' intuitive ideas and formal physics.
2. *The activation of resources is context sensitive* [6–9,37–39] where context includes any aspect of the environment that students notice [8]. This tenet creates an expectation of variability in student thinking. That is, we expect that resources will show up in different forms and at different frequencies in different contexts. Thus, observing a student use an idea in one context does not guarantee that we will observe that same student use the same idea in another (even similar) context, nor does *not* observing an idea in one context guarantee that we will *not* see that idea in another.
3. *Learning involves changing the structure or activation of resources*, by reorganizing, refining, properly activating, increasing the degree of formality of, or changing the role of resources [6–10,38]. For example, diSessa [9] theorizes that a primary difference between novice and expert cognition in physics is in the structure and connectedness of networks of resources. In this view, the resources that novices activate as they sense make about the natural world become part of the structures that organize expert physics thinking. The idea that resources are integral to learning is also reflected in the language that researchers use to describe them. That is, resources are often depicted as resources *for* something—for "understanding physical phenomena" [7], for "learning" [10], for "the development of a conceptual understanding of Coulomb's law" [37], for "thinking about physical situations" [6], or for "cognitive growth" [10]—emphasizing their generative role in thinking and learning.

These three tenets of resources theory shape our work. The expectation of sensibility and continuity with formal physics embodied by the first tenet directs our attention to patterns in student thinking that seem like "seeds" [24–26] or "conceptual progenitors" [23,35] of electric circuits concepts and prompts us to ask ourselves why a reasonable person might answer the way we observe students doing. The expectation of context sensitivity shapes our interpretation of patterns in student responses. Though we are searching for *common* resources for understanding circuits, we do not expect the resources we identify to be used in the same way across questions or students. Our work draws most from the *orientation* of the resources theoretical framework, which shapes the instructional relevance of our work; we turn to this next.

B. Instructional significance

Our team takes a pragmatic approach to research, seeking to produce work that has the potential to inform what instructors do in the classroom. Specifically, we aim to *shape instructors' expectations that students have generative ideas for their learning about circuits*. In identifying specific resources for understanding electric circuits, our research has the potential to shape instructors' (i) belief that students have generative ideas for understanding circuits, (ii) knowledge of common, generative ideas students use to reason about electric circuits, (iii) plans to teach in a way that elicits and builds from these resources, and (iv) recognition of these resources as they are deployed by students in real time. We expect our work will serve as *one input* to a complex process of both emergent and planned decision making.

Our focus on *common* conceptual resources is motivated by a model of generalizability that emphasizes recurrence across heterogeneous data sources [40]. In this model, heterogeneity increases predictive capacity; if a pattern is observed across a variety of different contexts, the assumption is that it is less likely to have been a context-specific (or random) effect. In our case, the extent to which we observe the same resource being used by many students in many contexts makes it more likely that other instructors may observe similar resources in similar (though not yet tested) contexts. This predictive capacity is limited by the representativeness of our sample, as we discuss in Sec. IV. To be considered common, a resource had to be used by at least 10% of students in at least one sample, and it had to be used by students in response to more than one question. Because much of our data come from student responses after some instruction, it is appropriate to consider these resources that may be leveraged *during* instruction, after students have had some exposure to circuits concepts.

As we described earlier, we are particularly interested in shaping instruction by providing knowledge of students' ideas about specific introductory physics topics that can complement existing misconceptions- and difficulties-oriented research. Typically, misconceptions and difficulties are either reported as an idea—e.g., students think of the battery as a constant source of current—or as something students find difficult (or something that is lacking in students' thinking)—e.g., students have difficulty applying the concept of current, or students lack a model for complete circuits. The grain size of these reported misconceptions or difficulties differs from the grain size of, for example, diSessa's [9] characterization of p-prims as small pieces of (phenomenological) knowledge, such as "bouncing" or "force as mover." In our efforts to complement misconceptions- and difficulties-oriented research, we choose to report conceptual resources at grain sizes comparable to characterizations within that research base, often describing resources in terms of "ideas," such as "current is responsive" or "voltage drives current." Our resources often express relationships between ideas and sometimes appear together; in this sense, they are not mutually exclusive, though they usually express distinct ideas—e.g., about current vs voltage.

This choice of grain size is consistent with other—but not all—resources reported in the literature, such as "the [less massive] car reacts twice as much" in a collision [6]. This choice is also informed by our methods and goals: to identify common conceptual resources that highlight the relationship between these ideas and the ideas that instructors want to develop—e.g., the resource "current is responsive," as stated, foregrounds the continuity between students' thinking and Ohm's law. Framing resources at higher levels of abstraction also affords the possibility of these resources showing up for many students in many contexts and thus has the potential to be generalizable in the way that many misconceptions and difficulties have proven to be. In the next section, we offer more detail about *how* we identify common conceptual resources for understanding circuits.

IV. CONTEXT AND METHODS

In this section, we describe our conceptual questions, our sample populations, and our data analysis methods.

A. Conceptual questions

For this study, we analyzed students' responses to five conceptual questions about circuits (Figs. 2–6): the order-of-elements question; the add-a-wire question; the compare-bulbs-A, B, C question; the modified-rank-the-bulb question; and the compare-bulbs-batteries-series question. Except for the order-of-elements question, these questions were modified from concept inventories or previous studies. We chose to primarily use (modified) existing questions for two primary reasons: first, already-existing questions have been tested and shown to be understandable to students, and second, already-existing questions afforded the best comparison between our results and ideas reported

A TA constructs two circuits (shown at right). Each circuit contains the same 3 bulbs and 3 batteries. The only difference between the two circuits is the order in which the elements are placed. The brightness of the bulbs in circuit A and B are the same. (*i.e.,* all 6 bulbs shine equally bright). Does this observation make sense to you? Explain how your understanding of circuits supports or opposes this observation.

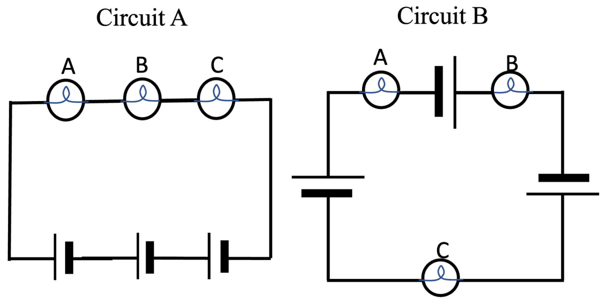


FIG. 2. Order-of-elements question.

When a wire is connection between points 1 and 2, as in the rightmost figure, the brightness of bulbs A and B does not change. How do you make sense of this? (We really want to know what makes sense to you, so if this *doesn't* make sense, say why not or what you expected differently.)

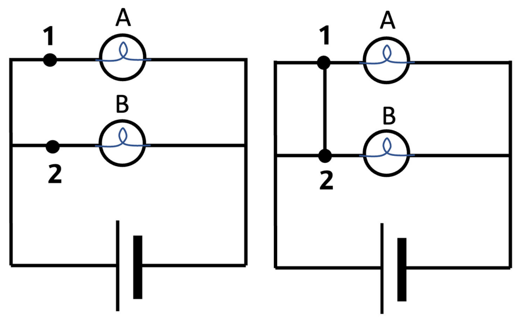
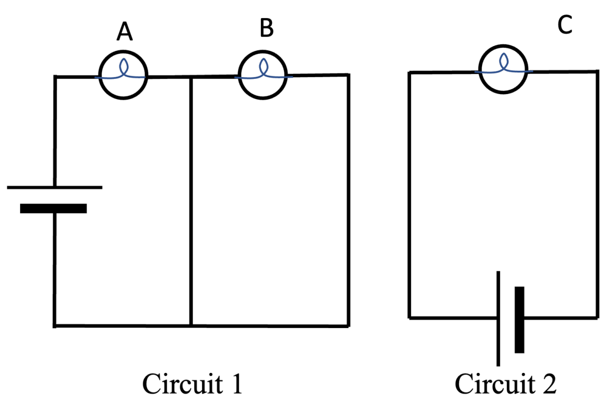


FIG. 3. Add-a-wire question.

in the existing literature. Though making comparisons between previously reported difficulties and the resources we report is not the purpose of this paper, using questions that have been used in prior studies puts our findings in more direct conversation with existing literature. In most cases, our modifications to previously used questions followed a method reported by Goodhew *et al.* [41]—transforming "predict" questions that ask students to make predictions about a particular scenario into "explain" questions that describe an outcome or phenomenon and ask students why that outcome makes sense to them. This kind of modification has been shown to produce questions that elicit more detailed and/or varied student reasoning, which suits our goal of understanding what may be fruitful in students' thinking. We also modified questions to clarify them or to narrow their focus toward specific observations or physics concepts.



Compare the brightness of bulbs A through C.
(a) Which bulb or bulbs are the brightest?
(b) Why does your answer make sense to you?

FIG. 4. Compare-bulbs-A, B, C question.

As part of a broader study, we collected student responses to more than 15 conceptual questions about circuits; the specific questions we report in this paper are a subset of those. We narrowed our dataset to include only questions for which (i) students tended to elaborate on their thinking, giving us more opportunities to understand their reasoning; (ii) the reasoning students provided spanned topics within circuits (i.e., we wanted ideas about current, voltage, resistance, etc.); and (iii) multiple institutions, with high response rates, and/or large numbers of students provided data. This last consideration generally played a primary role in determining which questions we report and is largely dependent on data collection limitations, like which questions our data collection partners selected.

1. Order-of-elements question

The order-of-elements question (Fig. 2) was created by our team and is meant to explore whether (and why) students think the arrangement of circuit elements matters.

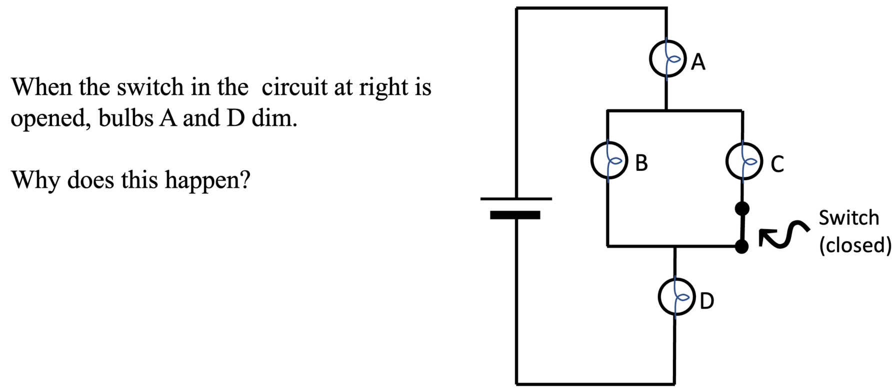


FIG. 5. Modified-rank-the-bulbs question.

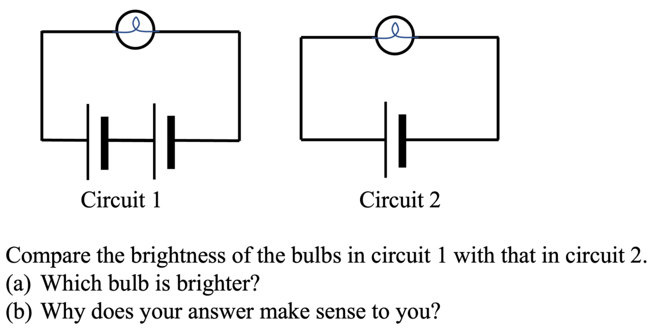


FIG. 6. Compare-bulbs-batteries-series question.

Students are told that circuits A and B differ only in the order of the bulbs and batteries that make up the circuit and that both circuits' bulbs are equally bright. Students are asked why this observation makes sense. One possible correct approach would foreground resistance and current; for example, one student in our dataset wrote: "Since each bulb and resistor are in series it doesn't matter what the order the series is in as the rule for current through a series does not depend on the order of the resistors or bulbs. Circuit A and B both have the same lightbulbs in a series so the total resistance and current is the same." Other correct responses approached the problem with an emphasis on voltage; for example, one student said "In circuit A, the voltage before the bulbs is 3 V but must divide evenly across the three bulbs resulting in a 1 V potential difference across each bulb. In circuit B, the voltage before each bulb is 1 V but is not divided since there is only 1 bulb after each battery."

2. Add-a-wire question

The add-a-wire question, adapted from Engelhardt and Beichner [15], presents two nearly identical parallel circuits, except that a wire is added in the second circuit across the parallel branches, on the same side of the bulbs relative to the battery (as in Fig. 3). This question asks students to explain why the brightness of the bulbs is the same in both circuits. This question was designed to encourage students to consider how the addition of new circuit elements may affect the circuit, if at all. A canonically correct explanation of this phenomenon expresses that adding a wire does not substantively change the potential difference across or resistance of the bulbs or battery, as in this student response: "Adding a wire in from 1 to 2 makes no difference in brightness because brightness is determined by current […] adding the wire does not affect current because V and R are not affected by the wire which has $R = 0$ Ohms."

3. Compare-bulbs-A, B, C question

The compare-bulbs-A, B, C (Fig. 4) question, adapted from Engelhardt and Beichner [15], asks students to compare the brightness of bulbs A and C in circuits 1 and 2, where circuit 1 is a parallel circuit with two bulbs—one in the main circuit with the battery and one in parallel with a wire—and circuit 2 is a simple series circuit with one bulb and one battery. This question was originally designed to check whether students would try to apply the heuristic that the current takes "the path of least resistance" [15]. In choosing this question for our study, we hoped to learn about how students think about current and resistance and expected that comparing a circuit in which current splits to one in which current does not (and where the resistances are different) might cue current and resistance reasoning.

In this scenario, bulbs A and C are equally bright and bulb B does not light. A correct explanation to this question includes that no current flows through bulb B (or the potential difference across B is zero) because the potential difference across or resistance of the wire between A and B is zero, so no current flows. This means the same current flows through A (or A has the same potential difference) as C. An example (correct) explanation given by a student was, "Circuit 1 shorts out bulb B, so no current goes through it because it can take a path with no resistance back to the […] battery. This creates two identical circuits so the brightness of A and C are the same."

TABLE I. Samples that received each written question.

	Sample			
Conceptual question	University	Context	N	Response rate
Order-of-elements	U1	Homework	316	88%
	U2	Homework	484	73%
Add-a-wire	U1	Homework	126	75%
	U4	Exam	30	100%
	U7	In class activity	26	70%
Compare-bulbs-A, B, C	U3	Homework	33	46%
	U4	Exam	58	98%
	U5	Exam	6	86%
	U6	Quiz	12	48%
Modified-rank-the-bulbs	U1	Homework	234	87%
	U3	Homework	55	47%
Compare-bulbs-batteries-series	U1	Homework	126	75%
	U3	Homework	33	46%
	U5	Exam	6	86%
	U6	Quiz	12	48%

4. Modified-rank-the-bulbs question

The modified-rank-the-bulbs question (Fig. 5), adapted from McDermott and Shaffer [18], was originally designed to identify the extent to which students think of a battery as a source of constant current, where current is independent of changes made in the circuit [15,18]. We selected this question for our study to understand how students make sense of the more complex four-bulb circuit in comparison to the simpler circuit that is created when the switch is opened (effectively three bulbs in series). In particular, we hoped it would cue ideas about the equivalent resistance of a network of bulbs or ideas about how multiple paths affect current flow. A canonically correct explanation of the phenomenon highlights that the resistance of the circuit as a whole changes when the switch is opened, which affects the current flowing from the battery or through bulbs A and D. The following response illustrates one way a student might connect these ideas: “When the switch is opened, the current no longer has the option to split and flow through either B or C, it must all flow through B. Since B and C are no longer in parallel, the overall resistance of the circuit is greater than when the switch is closed. With greater resistance, there is less current according to Ohm’s law. And since the luminosity of a light bulb is directly proportional to current, this means dimmer light bulbs as well.”

5. Compare-bulbs-batteries-series question

The compare-bulbs-batteries-series question (Fig. 6), adapted from Engelhardt and Beichner [15], asks students to decide whether a bulb in a series circuit with two batteries will be brighter than a bulb in a series circuit with a single battery. This question gives students the opportunity to connect the ideas of voltage, current, power, and brightness. A correct answer to this question states that the bulb in circuit 1 is brighter because more current flows through it or there is a greater potential difference across it, as in this student response: “Potential of the batteries add in series for the total voltage of the circuit. This means [bulb] 1 will have higher voltage, but consistent R, which results in higher current ($V = IR$).” Note that bulbs are not ohmic resistors, so even if the bulbs are identical, they may not have the same resistance because the potential difference across each is different. This answer would be accurate for ohmic resistors, and we would count it as correct in this situation because it correctly states the effect of adding a battery.

B. Sample and data collection

We collected and analyzed 1557 responses to the questions above, from students in introductory algebra- and calculus-based physics courses at seven colleges and universities in the United States. Response rates from these institutions ranged from 46% to 100%, depending on the context (Table I). Conceptual questions were included on in-class work, homework, quizzes, and exams at different institutions, creating a context for multiple sources of heterogeneity [42]. Table II in the Appendix describes the context of each institution in more detail.

The ethnoracial demographics for the colleges and universities in our study versus all undergraduates enrolled in degree-granting institutions in the United States is shown in Fig. 7. Figure 7 suggests that the universities in our study sample are not racially or ethnically representative of the overall population of undergraduates, similar to the trend present in physics education research of oversampling from white, wealthy, high-mathematics-SAT-scoring student

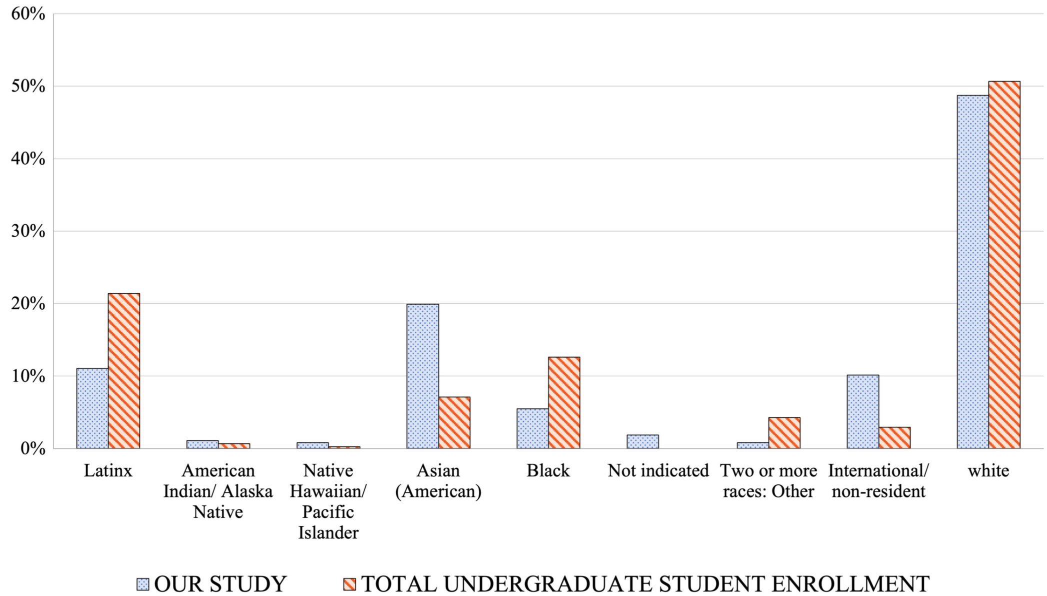


FIG. 7. Ethnoracial demographics of the institutions in our sample (blue dotted) versus all undergraduates enrolled in degree-granting institutions (orange diagonal lines). The blue bars were constructed using demographic data provided by offices of institutional research or instructional websites, weighted by sample size. The orange bars were constructed using data from the National Center for Education Statistics [45].

populations [43]. The universities in our study sample serve a disproportionately higher fraction of Asian (American) students and international students, and a disproportionately lower fraction of Latinx and Black students. In addition, the median parental income of the students at colleges or universities in our study is higher than the national average. Thus, the demographic composition of our sample may limit the extent to which our results are applicable to a representative sample of introductory physics students.[1,2,3]

C. Data analysis

Data analysis was qualitative in nature [42] and informed by a resources orientation toward student thinking, foregrounding student ideas that we consider to be generative for physics learning, in the sense that we can imagine instructional sequences that build from these ideas toward more sophisticated or canonical understandings [7,9,10,46,47]. For example, the idea that "friction slows things down," while not technically correct in all circumstances, is an expression of the broader notion that "forces affect the motion of objects," and could be the starting point of an instructional sequence that generalizes toward Newton's second law [47]. In keeping with our focus on common resources, we report only those resources that we observed in at least 10% of responses in a sample, and that were evident in students' responses to multiple questions. Our characterization of resources focuses on commonality (not comprehensiveness); there are resources that students in our sample used to reason about circuits that we do not report here. Our goal in characterizing ideas is not to capture every possible resource, or to come up with the same set of resources that other groups of researchers would, but to identify a set of resources that are common and that other, independent observers could also perceive in our data [48].

To identify resources from student responses, we first characterized what we considered to be "seeds of science" [24] in students' answers to the conceptual questions in Sec. IV. We then identified themes in these "seeds of science" across questions, in order to construct a set of *common* resources, which then formed the basis of our

[1]Forthcoming work from our team will discuss limitations in how ethnoracial demographic data are collected and reported, and some of the ways in which this shapes what we can know about the demographics of PER. For example, most universities report demographic data under an aggregate category of "Asian or Asian American," while the literature extensively calls for the disaggregation of data for Asian(American) students, arguing that "some subgroups may not be accurately represented by their common aggregate classification" [44]. For instance, the experiences of groups who immigrate to the United States to escape genocide in their home countries are often very different than the experiences of groups who immigrate to the United States to pursue jobs in the tech industry.

[2]Here we report only ethnoracial demographic data, but there are additional axes of domination in physics—and associated minoritized groups—that we do not report. This is partly a limitation in the data available—e.g., we do not have aggregate statistics on disability status—and partly due to ethical conundrums, we face in reporting some data—e.g., the National Center for Educational Statistics, our source for national data, reports gender demographics solely in terms of male and female students. These dilemmas are central to a separate strand of our team's work.

[3]Our analysis focuses on data from the United States.

emergent coding scheme [49]. This method is consistent with methods developed to identify common difficulties or misconceptions; we intentionally mirror these methods so as to contribute complementary insights. (This process is described in more detail in Ref. [50].)

To illustrate this process, consider the following example responses to the order-of-elements question:

- "It doesn't matter the order of the elements are placed in as long as the circuit configurations don't change, the current in series will be constant throughout."
- "Current is the same throughout a closed series loops."

In a first-pass analysis of this question, we might flag that the first response includes the productive idea that *the order of elements does not matter as long as the configuration does not change*, and that both responses include the productive idea that the *current is the same throughout a closed series loop*. Continued through the data, we also note responses like:

- "The circuit shows A and B in parallel in both scenarios, so the brightness of the bulbs would not change regardless." [add-a-wire question]
- "In series, resistance goes up, in parallel it goes down…" [modified-rank-the-bulbs question]
- "When batteries are in series their voltages stack to create a much larger potential difference." [compare-bulbs-batteries-series question]

Again, in a first pass through these responses, we might note that the first response includes the idea that *the connections in the two circuits are the same so brightness of the bulbs is the same;* the second response includes the idea that *resistance depends on the connections between elements (in series resistance goes up and in parallel it goes down);* and the third response includes the idea that *voltages add in series.*

Though we could categorize these ideas in a variety of different ways, we would claim that *all of them* relate to how elements are connected in the circuits and what that "means" for properties of the circuits (e.g., current, voltage, resistance, and brightness). Notably, this commonality connects to a central goal in introductory physics instruction about circuits: that *the arrangement of circuit elements matters for the properties of the circuit.* This commonality then became one of the resources in our coding scheme.

We chose to construct resource categories from our data, informed by our understandings of introductory physics course goals, rather than to impose an external coding scheme based on, for example, reframings of existing literature on student thinking about circuits. This choice is guided in part by an open question as to whether applying a resources lens to data will generate altogether new insights about student reasoning about circuits than a reframing of findings based on a misconceptions or difficulties lens. In our previous research, we have found that our characterizations of resources sometimes overlap with (and could be thought of as reframings of) existing categories in the literature but always include new ideas that have not yet been reported [47].

After clustering a number of different ideas into a set of resources, authors L. C. B. and B. H. applied the resulting coding scheme to 10% of the data, and then iteratively refined the coding scheme until an agreement of over 95% was reached. L. C. B. and B. H. then independently coded the entire dataset using the finalized coding scheme. In this process, a single student response could be given zero, one, or multiple codes, depending on the number of resources used. After L. C. B. and B. H. coded the entire dataset, percentage agreement between coders was calculated by taking the normalized difference between the total number of codes and the total number of disagreements between the two coders:

$$\frac{(n_{\text{possible codes}})(n_{\text{coded responses}}) - (n_{\text{total disagreements}})}{(n_{\text{possible codes}})(n_{\text{coded responses}})}$$

Percentage agreement was used over a standard statistical measure like Cohen's kappa because most standard measures require that individual codes be independent and mutually exclusive [51], neither of which is true of our resource codes. The overall percentage agreement for the full dataset was 94%. The percentage of responses that were assigned to each resource code (reported in the following section) reflects only codes that both coders agreed on; this is a conservative estimate and may underrepresent what some researchers would identify in the same data.

V. RESULTS

In this section, we present some of the common conceptual resources for understanding circuits that we identified in our dataset. The frequency of the resources that we observed in student responses to each question is presented in Sec. V.C.

A. Ohm's law trio: Current is responsive, voltage drives current, resistance limits current

We report these resources as a set (the "Ohm's law trio") because they were frequently used together to make sense of the relationship between current, voltage, and resistance. We named these resources "current is responsive" (to changes in the circuit), "voltage drives current," and "resistance limits current," to capture both (i) the meaning that students seemed to be making of these quantities and (ii) the ways in which we perceive these ideas to be fruitful for scientific reasoning about electric circuits. The "current is responsive" resource sometimes appeared alone, but the "voltage drives current" and "resistance limits current" resources always co-occurred with the "current is responsive" resource since these two resources express a relationship between voltage or resistance and current. A single response could receive (a) only the "current is responsive" code, (b) both the "current is responsive" *and* "voltage drives current" codes, or both the "current is responsive" *and* "resistance limits current" codes, (c) all three codes, or (d) none of these codes.

1. Current is responsive

The "current is responsive" resource captures the idea that the amount of current that flows in a circuit, branch, or element is influenced by other circuit elements. We assigned the "current is responsive" resource code to any responses that treated current as a dependent variable that responds to differences or changes in circuits, such as changes to resistance, voltage, or the arrangement of elements. Students referenced a number of ways that current could "respond," including the direction that current travels, the total amount of current in a branch or location, and the flow rate of charges (or even the "speed" of charges or current) in the circuit. Both positive and negative statements (i.e., "*Current changes because x changes,*" and "*Current doesn't change because x doesn't change*") were assigned this resource code. Typical examples include:

> […] When the switch is closed it adds another path for current to flow. This increases the total current so A [and] D would be brighter. Since the switch is open, the opposite would happen, making A [and] D dimmer. [modified-rank-bulbs question, Fig. 5]

> The potential difference is twice the amount in circuit 1 than in 2, so twice of the amount of current has to be passing in if both bulbs have equal resistance. [compare-bulbs-batteries-series, Fig. 6]

The fruitful idea we highlight in the first response is that opening or closing a switch changes the current in the entire circuit: "add[ing] another path for current to flow… increases the total current [through bulbs A and D]… [when] the switch is open, the opposite would happen." The second response recognizes that resistance affects current flow by specifically stating that the conclusion is true [the current increases] if the bulbs have identical resistance. The fact that the current supplied by a battery is not fixed or constant is a challenging and important idea for introductory physics students to understand. This resource is one idea that may be generative for learning more sophisticated models of electric circuits. We observed responses consistent with this resource for every question we asked and in 20% of all responses. We discuss the frequency of each resource across each question in more detail in Sec. V C.

In some cases, students used the "current is responsive" resource in answers that aligned with previously reported difficulties or misconceptions. For example, some responses that we assigned the "current is responsive" code used ideas that may be consistent with the "sequential reasoning" misconception named in the literature—the idea that current "approaches" and is affected by each element in sequence rather than to the arrangement of and elements within the circuit as a whole. Two examples are as follows:

> […] In both circuits, the batteries have the same orientation so the current will flow in the same direction throughout both circuits. The bulbs in both circuits can represent the drop in total potential and batteries represent the gain in total potential in the circuits and since they equal each other, the brightness of all bulbs will be the same because the same amount of the total current will run through each bulb of each circuit. [order-of-elements question, Fig. 2]

> […] In circuit A, the batteries are all in a row which increases the total current but the resistors are also in a row which would decrease the total current. Circuit A has 3 times the voltage but also 3 times the resistance, which would result in the same current. Circuit B also has the same current because the voltage is one and the resistance is 1, but it happens 3 times in series. [order-of-elements question, Fig. 2]

In the first response above, the "current is responsive" resource is demonstrated by the statement that the current is the same in the two circuits of the order-of-elements question because the potential difference across the identical bulbs and batteries in each circuit is the same. This is an appropriate way to think about the question. At the same time, the first sentence of this response ("the batteries have the same orientation so the current will flow in the same direction throughout both circuits") is reminiscent of the idea that the direction of current matters, which could be interpreted as sequential reasoning [33]. Likewise, the second response could be read as using sequential reasoning, because it focuses on the order of the elements and names the impact of individual elements separately. However, both responses rely on the generative idea that current is affected by the equivalent potential difference and/or resistance of the circuit.

We see the "current is responsive" resource as a fruitful "seed" of a sophisticated scientific understanding that the current is dependent on the resistance of, and potential across, circuit elements, branches, or networks. This resource could support students in predicting that changing the resistance of a circuit element or adding or removing a resistor from a branch affects not only the current in that branch but also in other parts of the circuit as well. In light of McDermott and Shaffer's sense that "perhaps the most pervasive and persistent difficulty that students have with dc circuits is the belief that the battery is a constant source of current" [18], this resource is worth instructors' attention even if it seems rather basic or general. This resource can be highlighted, further explored, and refined to support students in developing accurate conceptual models for electric circuits.

2. Voltage drives current

As the name suggests, the "voltage drives current" resource captures the idea that voltage or potential difference causes current or charge to flow in a circuit or element. This resource showed up in 9% of all responses and was particularly common (29%) in responses to the compare-bulbs-batteries series question. With this resource, students reason that a potential difference (or voltage, emf, and other terms chosen by students) makes current flow through the circuit and that more potential difference means more current, maintaining the potential difference means the same current, and so on. For example,

> [In circuit 1] the emf's add together and should generate a larger current. [compare-bulbs-batteries-series, Fig. 6]

> […] An extra battery increases the voltage which increases the current which is what makes the light bulb [in circuit 1] glow brighter. [compare-bulbs-batteries-series, Fig. 6]

> There is zero potential difference between points 1 and 2, $V1 = V2 = V$. Thus, there is no current flow between points 1 and 2. [add-a-wire question, Fig. 3]

> […] Shorting point 1 and 2 does nothing to the circuit because there's no voltage across 1 and 2. No current flow between [1] and 2 since there's no potential difference. [add-a-wire question, Fig. 3]

Each of these examples makes a connection between voltage or potential difference and current flow, which is an important idea in a canonical model of electric circuits and is particularly appropriate for circuits with a number of identical resistors, as in our questions. The example responses imply varying degrees of causality—for example, the language "to create" implies that the potential difference takes a more active role than "therefore." Yet, they demonstrate the essence of the resource—the potential difference across an element, branch, or network affects the current flow in a semimechanistic way. In the first two example responses, students answered that a circuit with two batteries and one bulb has more voltage (or emf) and thus more current than a circuit with one battery and one bulb. In the final two responses, students reasoned that if the potential difference between two points is already zero, adding a wire connecting these two points will not affect the behavior of the circuit because no current will flow across that wire. This argument is not entirely accurate; ideal wires carry current when there is a potential difference across adjacent circuit elements, despite the fact that there is no potential difference across an ideal wire because it has no resistance. In this situation, then, the current would split evenly between the two wires connecting points 1 and 2. Still, the ideas expressed in these examples lead to the same conclusions about the behavior of the bulbs and battery in the circuit. More importantly for our analysis, the idea that the potential difference causes current to flow in a branch or element is relevant and fruitful for reasoning about circuits like these —particularly for real wires and elements.

This idea may be fruitful for connecting students' reasoning about the behavior of electric circuits to concepts of electric potential, field, and charge motion that they have explored in other physics contexts, and it could support the development of a causal model for the behavior of electric circuits which the literature reports as a difficulty for many students. In particular, the literature emphasizes that many students think current is the primary, independent variable and confuse cause and effect [13]. Our findings point to contexts in which students articulate a connection between voltage and current—one that identifies voltage as a primary factor in affecting the current. Instructors could leverage this resource by drawing out connections to students' understandings of electric potential, field, and force in other contexts, or by supporting students in examining the relationship between potential difference and other electric circuit properties like resistance.

3. Resistance limits current

Student responses often explained that resistance (of light bulbs, resistors, or networks) limits, slows, or impedes current in a circuit. We coded responses like these as instances of the "resistance limits current" resource, the essence of which is to describe an inverse relationship between resistance and current in a circuit or circuit element. This resource code was assigned to 12% of responses in our dataset.

Some responses we assigned the "resistance limits current" code focused on the current's response to resistance, describing the current as "overcoming" obstacles to travel through the circuit, or "wanting" to avoid resistance. For example, some students answered the compare-bulbs-A, B, C question (Fig. 4) with the correct ranking ($A = C \gg B$, or A is as bright as C which is much, much brighter than B) and justified their response by explaining that B will not light because the "current wants to take the path of least resistance" and therefore "bypasses" B such that A and C are effectively the same circuit. This kind of response draws on the resource "resistance limits current" to explain that the current does not flow in a branch with higher resistance when there is an available parallel branch of effectively zero resistance. This specific variation of the "resistance limits current" resource is particularly productive for the compare-bulbs-A, B, C question because the AB circuit in the question has one branch with nonzero resistance and one branch with zero resistance. In other situations, the idea that (all) current takes the path of least resistance would be incorrect. Still, we see this idea as worth instructors' attention because it captures valuable intuition that can be leveraged in instruction to help students develop their understanding of how resistance affects the flow of current in various branches or networks of a circuit.

Other responses that we coded as "resistance limits current" referenced Ohm's law to characterize the relationship between current and resistance, as in these examples:

> […] According to Ohm's law, the current decreases when the resistance increases (V is the same)… [modified-rank-the-bulbs, Fig. 5]

> […] When the switch opens, bulb B joins the series instead of just being in parallel to bulb C, so resistance increases. The battery is the same so V remains the same. Therefore current must decrease to compensate. [modified-rank-the-bulbs, Fig. 5]

Both of these responses, and others like them, express the important idea that when voltage is held constant, more resistance in a circuit means less current flows. Although light bulbs are not ohmic resistors, the qualitative relationship between current and resistance that these responses express is appropriate and fruitful.

Some responses that received this code stated that fewer paths and/or more bulbs/resistors in a circuit means less current in the circuit, as in this response to the modified-rank-the-bulbs question (Fig. 5):

> When the switch is opened, the current no longer has the option to split and flow through either B or C, it must all flow through B. Since B and C are no longer in parallel, the overall resistance of the circuit is greater than when the switch is closed. With greater resistance, there is less current according to Ohm's law. And since the luminosity of a light bulb is directly proportional to current, this means dimmer light bulbs as well.

A key idea in this response is that with fewer paths, the resistance of the circuit is greater and therefore it carries less current. Physics instructors may argue that light bulbs are not ohmic resistors, and therefore Ohm's law is not quantitatively applicable, and we agree. However, we believe that the idea that "with greater resistance, there is less current" is appropriate and fruitful for making sense of the circuit in question. This is a more complex instantiation of the "resistance limits current" resource, used in conjunction with the idea that "more paths means less resistance" (a specific version of the "connections" resource we discuss below). Examples like this one illustrate how the "resistance limits current" resource might be combined with other resources to deploy an accurate "paths and obstacles" model for electric circuits like the one developed in the Physics by Inquiry curriculum [52].

We argue that the "resistance limits current" resource, in its various instantiations, may be fruitful for developing a conceptual model of the behavior of current flow in battery-resistor circuits, or for making sense of Ohm's law and equivalent resistance rules. Instructors may leverage this resource by encouraging students to consider resistance and current in limiting cases like the circuits in the compare-bulbs-A, B, C question or in more complex networks of bulbs or resistors in parallel and in series.

4. Ohm's law trio revisited: Using the three resources together

When used together, the three resources in the "Ohm's law trio" form the beginnings of a conceptual model for the behavior of current in circuits (though, with resources theory as a lens, we would not expect this conceptual model to be coherently or consistently deployed). That these three resources were each used somewhat commonly by the introductory physics students who participated in this study suggests that these ideas may be readily available for physics instructors to elicit, connect, and build with. In fact, some responses in our dataset used these three resources together in what we recognize as a conceptual model for an electric circuit. For example, to the order-of-elements question (Fig. 2), a student wrote:

> […] Since electrons flow as a stream through the wire, the positioning of the components doesn't matter in this case. The "stream" is being slowed down by the bulbs, and pushed by the batteries, doesn't matter where. In tug of war for example, it doesn't matter much where you position the people pulling, as long as they are somewhere on the rope.

This response treats current as responsive to bulbs or resistors (it is "slowed down" by them) and batteries (it is "pushed" by them), and not responsive to the order of the resistors and batteries in the circuit, drawing on all three resources at once to model the movement of electrons as analogous to the movement of a rope in tug of war (where the placement of the batteries or people doesn't matter).

Other examples that connected current, resistance, and voltage include:

> [It] makes sense [that all the bulbs are the same brightness] because the current is the same everywhere in the wire and is being pushed forward by the same amount of voltage from the batteries and resisted by the same amount of resistance. [order-of-elements question, Fig. 2]

> […] The circuits have the same net voltage. The bulbs are all the same and due to their series nature, the equivalent resistance over both circuits is the same (the sum of the three). Therefore the current that runs through both circuits is the same. [order-of-elements, Fig. 2]

Both of these responses use the "current is responsive" resource to imply that the current is affected by the arrangement and number of bulbs and batteries in the circuit, the "resistance limits current" resource to note that the light bulbs or resistors affect the current, and the "voltage drives current" resource to explain how batteries affect the current. These responses illustrate one way in which the "Ohm's law trio" resources may be generative for students: they are useful for articulating a model for what is happening in a circuit. We do not mean to say that the models embedded in the responses that we assigned all three codes are complete or useful for all circuits; rather, these represent generative ideas that instruction can leverage. We will describe how we, as a project, are pursuing that in the Discussion section.

B. Connections resource: The way elements are connected within the circuit matters

Responses in our dataset frequently used the idea that the way elements are connected (i.e., the specific arrangement or orientation of elements) impacts the behavior of the circuit as a whole. This includes ideas about how the arrangement or connections of elements (including ideal wires) affect or determine resistance, voltage, or current through a network of elements. We grouped these ideas as the "connections" resource, which we assigned to 37% of responses across our dataset. For example:

> All elements are in series for both cases, and they all have the same connectivity in terms of the pos. and neg. terminals of the batteries and bulbs. According to the loop rule: $V_A + V_B + V_C = V_{\text{batteries}}$ in both cases. And for the same loop, the current that runs thru [sic] it remains constant unless it reaches a junction. Thus, all bulbs glow equally bright." [order-of-elements question, Fig. 2]

A central idea in this response is that the structure of the whole circuit—the junctions, elements in series, and arrangement of batteries—affects the current through or voltage across circuit elements. This is the essence of the "connections" resource.

A common instantiation of this resource was the idea that the series or parallel arrangement of bulbs in a circuit affects the brightness of bulbs or current flowing through them. In some cases, students used this reasoning to argue that changing the arrangement of the circuit elements would change the voltage, current, or resistance. In other cases, students used this reasoning to argue that if a set of changes (or differences between two circuits) resulted in an equivalent arrangement of the elements, the resistance, current, or voltage would be the same. For example, the response below used the "connections" resource to explain why opening the switch after bulb C in the modified-rank-the-bulbs circuit results in bulbs A and D to dim:

> [Bulbs A and D] dim because the resistance in the circuit has increased sin[c]e B and C are no longer in parallel with each other. Items in parallel have a lower equivalent resistance than if they are in series. [modified-rank-the-bulbs question, Fig. 5]

This response argues that opening and closing the switch in the four-bulb circuit changes the structure of the circuit in a way that affects the equivalent resistance of the set of bulbs. The "connections" resource, as illustrated here, captures the dependence of the behavior of the circuit on specific attributes of its structure. In many cases, this particular instantiation of the "connections" resource is an articulation of global reasoning, as students are emphasizing how a local change (e.g., opening the switch) impacts the circuit overall (e.g., increases equivalent resistance). Although the literature reports global reasoning as challenging for students—many opting to employ local reasoning where they do not recognize that a change in one part of the circuit impacts the circuit overall [13,21,22,32]—we find it promising that this resource showed up in almost 50% of responses to the modified-rank-the-bulbs question.

In other examples of this resource, students explained that some arrangements are equivalent:

> The voltage of a circuit in series is equal to the sum of all voltages. Nowhere in this rule/law does

> it state what the geometry or setup of the circuit must be, only that it is in series. Seeing as the circuit is in series, I can find the total voltage, and I can also assume that the current is the same everywhere in a series circuit, this is not dependent upon arrangement of items. Thus, the bulbs will burn the same no matter how they are arranged, as long as it remains in series. [order-of-elements question, Fig. 2]

> Circuit A and Circuit B are both consisting of light bulbs connected in series—or, rather, resistors in series. We know that with resistors in series, the equivalent resistance will be the sum of all resistors. Therefore, in this case, it won't matter the order in which the resistors and batteries are connected as long as they stay in series with each other. [order-of-elements question, Fig. 2]

> Adding a wire to the circuit does not change the brightness because the bulbs A and B are still in parallel which means equal voltage from the battery is powering the identical bulbs. [add-a-wire question, Fig. 3]

The first response above describes the "total voltage" of the order-of-elements (series) circuits as a sum of the voltages, drawing on the "connections" resource to highlight that the bulbs will have the same brightness if they (and the batteries) are arranged in series. Although we cannot assess whether this response is technically correct because it is unclear what "sum" in this response refers to—e.g., all of the elements or just the batteries—a resources analysis foregrounds the fruitful pieces of this student's thinking. That is, in a resources analysis, the answer need not be canonically correct for instructors to identify and leverage the generative ideas therein—in this case, the way the elements are connected matters for the voltage and the current. The second and third responses accurately identify that the order of bulbs, batteries, and wires does not matter for voltage and equivalent resistance so long as they are connected in the same way, in this case, in series or parallel.

The "connections" resource also includes other types of connections or structures such as complete or closed loops, the number of loops or paths, and the orientation or direction of the batteries in a circuit. For example, students wrote:

> I mean a circuit is a loop, and using Kirchoff's rules, I know that everything essentially evens out when you look at all the elements in a closed loop, and given that both circuits are in closed loops, and have the exact same elements in them (light bulb resistance and voltage in number of batteries), you should get the same current, which equates to equal brightness when the light bulbs have equal resistance. [order-of-elements question, Fig. 2]

> […] [I] know current is the same everywhere in a single loop circuit, so this problem is pretty easy to intuit. [order-of-elements question, Fig. 2]

In these responses, students treat connections in a loop as significant for the deployment of particular rules about current and voltage. For example, students described how in single loops, they knew certain things to be true: "everything evens out," "current is the same," and "current remains constant unless it reaches a junction." While these responses contain ideas that are less commonly mentioned, the theme of the circuit's behavior being affected by changes to the arrangement or orientation of its elements is an underlying part of the reasoning given in each response.

These responses—particularly to the order-of-elements question—emphasize instances where students are explicitly *not* using sequential reasoning. Sequential reasoning—where students reason as though the direction of current and order of elements matter for what happens in the circuit—is another common misconception reported by the literature [18–21,27,30–34]. Our findings add nuance to the corpus of literature, suggesting that in at least one context (e.g., the order-of-elements question), students frequently consider how the arrangement of the circuit affects variables like current, voltage, and resistance. In the next section, we discuss the frequency of all these resources in more detail.

C. Frequencies of resources in our sample

Resources theory posits that resource activation is context-sensitive and based on a host of complex factors including the classroom environment and the question at hand. Therefore, we expect variation (even significant variation) in the frequency with which students use these resources across questions and universities. In this study, the frequencies of student responses that we coded for each resource are best interpreted as establishing that these resources are common and illustrating that their use varies, rather than as predictive of the specific fraction of students who may use each resource in other contexts. We hypothesize that these same resources may be *available* to students in other contexts (even if at different frequencies) and that the questions that more frequently elicited particular resources in our study might be more likely to elicit them in other contexts as well. We reiterate that these results likely do not come from a representative sample of introductory physics students (though we do not know, as a community, what a representative sample

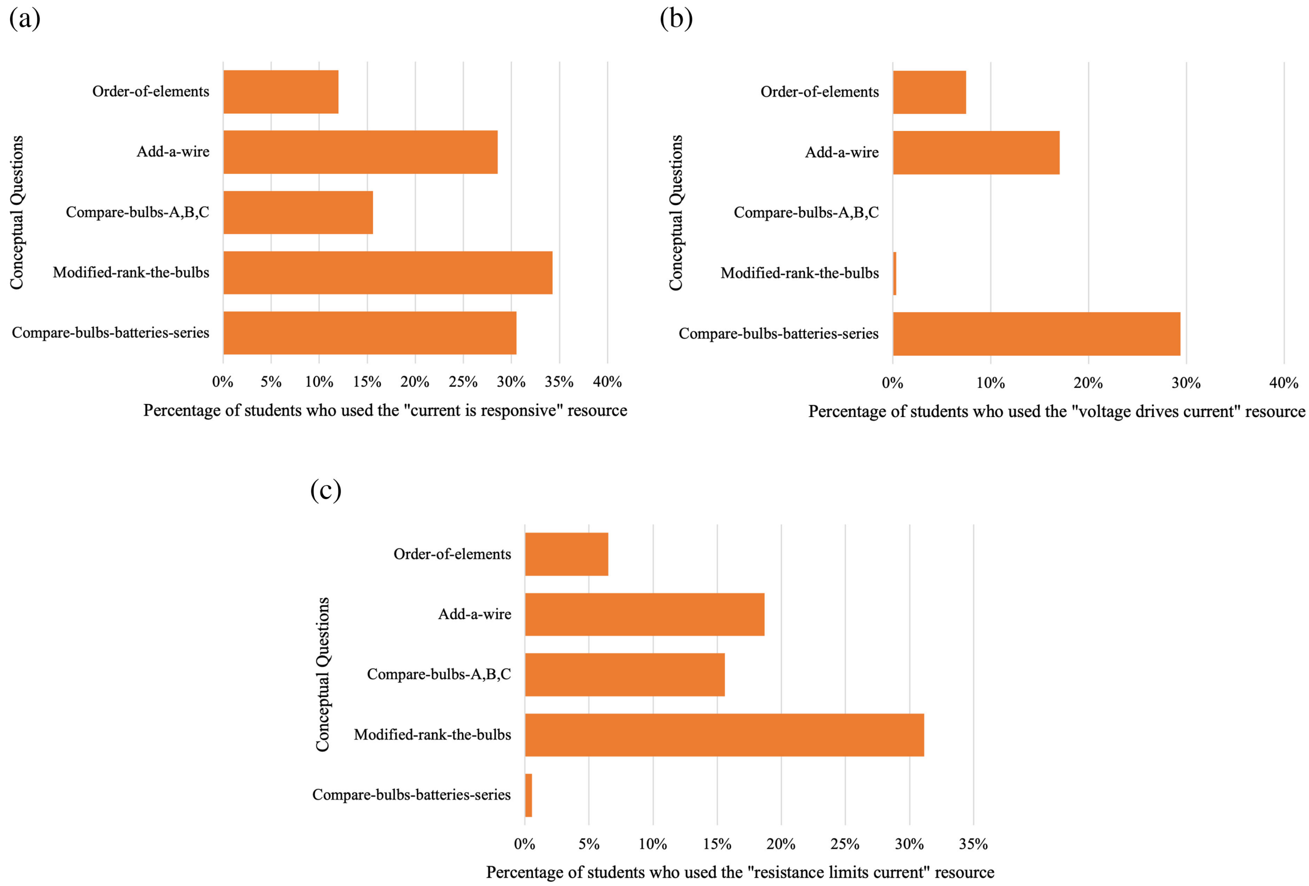


FIG. 8. Frequency of “Ohm’s law trio” resources: (a) “current is responsive,” (b) “voltage drives current,” and (c) “resistance limits current” by question.

would be [53], which limits the generalizability of our findings.

1. Ohm’s law trio

Figure 8 gives the percentage of responses to each question that we coded as including each of the “Ohm’s law trio” resources. These percentages reflect the fraction of all responses to each question that were assigned each resource code by both coders. Every question was administered at multiple universities and there was some variation in the frequency of this resource from sample to sample. Tables III, IV, and V in the Appendix give a detailed breakdown of the fraction of responses assigned the three "Ohm's law trio" resources for each questions and university.

As reflected in Fig. 8, we assigned the “current is responsive” code to between 12% of responses (for the order-of-elements question) and 34% of responses (for the modified rank-the-bulbs question). In this sense, the “current is responsive” resource was somewhat common in responses to all of the questions we used in this study. The “voltage drives current” and “resistance limits current” resources were common in some, but not all, questions. In particular, the “resistance limits current” resource was assigned to 31% of responses to the modified-rank-the-bulbs question, and very few responses to the compare-bulbs-batteries-series question. Conversely, the “voltage drives current” resource was assigned to very few responses to the modified-rank-the-bulbs question and nearly 30% of responses to the compare-bulbs-batteries-series question. This pattern may suggest that students implicitly interpret some questions, like the modified-rank-the-bulbs question, as “resistance questions” and other questions, like the compare-bulbs-batteries-series question, as “voltage questions.” If this pattern generalizes, it may help instructors select questions based on the kinds of resources they want to elicit.

2. Connections resource

As shown in Fig. 9, the “connections” resource was used in responses to all five questions in our study.

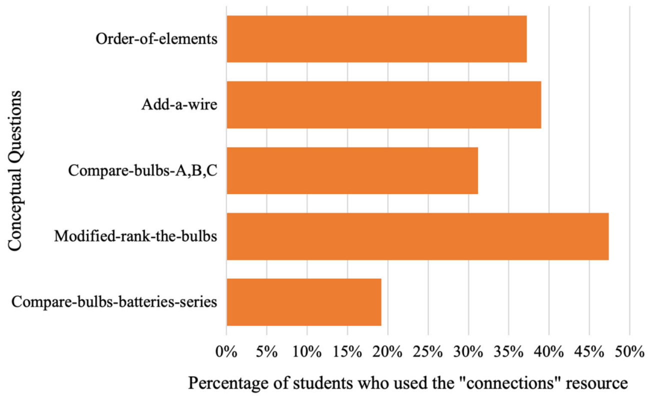


FIG. 9. Fraction of responses in our dataset that were assigned the “connections” resource code by question.

We assigned this code to between 19% of responses (for the compare-bulbs-in-series question) and 47% of responses (for the modified-rank-the-bulbs question). Every question was administered at multiple universities and there was some variation in the frequency of this resource from sample to sample. For example, no responses to the compare-bulbs-batteries-series questions from U5 used the “connections” resource, while approximately 25% of responses to the same question from U1 used this resource. (See Table VI in the Appendix for the detailed breakdown of frequency by question and university.) Whether this variation is due to the course curriculum or some other factor, we do not know. As above, the purpose of our analysis is not to predict the fraction of students who will use or have access to any of the resources we report for any particular course; rather, it is to provide instructors and researchers with a sense of some of the fruitful ideas we observe students using in our sample and that may be resources for understanding electric circuits more broadly. With this goal in mind, the patterns that we wish to highlight are (i) that this resource is relatively common across a range of question contexts, and (ii) that it appears to be more commonly elicited by some questions, like the modified- rank-the-bulbs question, which draws attention to the effects of the structure of a circuit on the brightness of the bulbs. One way instruction could support students to build from this resource is to first elicit it using a question or set of questions like the modified-rank-the-bulbs question, then encourage students to extend and test their ideas by applying them to a more complex circuit with parallel and/or series networks.

VI. DISCUSSION AND IMPLICATIONS

In this paper, we have reported some of the common conceptual resources for understanding electric circuits that we inferred from our data: the “Ohm’s law trio” resources, which includes “current is responsive,” “voltage drives current,” and “resistance limits current,” and the “connections” resource. Students from multiple universities used these resources in response to a variety of conceptual questions and in answers that were both canonically correct and incorrect.

Existing literature on student ideas about circuits emphasizes, in large part, student difficulties with and misconceptions about current. A subset of this literature represents student thinking in terms of models; that is, some literature suggests that students have relatively robust, coherent models for current that they deploy as they reason about circuits. For example, Shipstone *et al.* [21] suggest that students model current as a material thing that comes from the battery and gets used up as it travels through circuit elements. The same authors describe sequential and local reasoning—where students make predictions based on individual circuit elements and their placement rather than on the basis of the arrangement and composition of the circuit as a whole—as barriers to correct understanding. In contrast, our work demonstrates that students use a variety of fruitful ideas for understanding circuits that may support them in constructing models of electric circuits that are accurate and appropriate for introductory physics. The “connections” resource stands in contrast to local and sequential reasoning about circuits, and this idea in conjunction with the “Ohm’s law trio” resources provides beginnings of a scientifically accurate model of electric circuits. These findings suggest that students have a number of generative ideas available for making sense of electric circuits, and that instruction can help students to refine these ideas toward more general and powerful models.

Our team is using the results described here to develop instructional materials (e.g., worksheets) that elicit and build on the resources we have identified. We call these materials ACORN (Attending to COnceptual Resource iN) Physics Tutorials [54], and they serve as proof of concept that instruction can begin with knowledge of student resources for understanding circuits and leverage these resources to support students in articulating, applying, and testing their science ideas.

To illustrate how this has worked for us, we briefly describe the development and implementation of early versions of the ACORN Physics Tutorial on circuits. The focal question of this worksheet is the modified-rank-the-bulbs question, used extensively in the research we describe in this paper. The analysis we present above shows that this question frequently elicits the “current is responsive,” “resistance limits current,” and “connections” resources. In the ACORN Physics Tutorial on circuits, we use additional questions (not featured in the present study) to elicit the “voltage drives current” resource and to guide students to connect, extend, and test these ideas using real bulbs, batteries, and wires and/or simulations like the PhET DC Circuit Construction Kit [55]. A distinguishing feature of this tutorial—and other ACORN Physics Tutorials—is that it does not closely scaffold students toward a single, canonical model. Instead, ACORN Physics Tutorials encourage students to write down models based on their own observations and using their own conceptual resources. One of the primary motivations behind these design choices is to illustrate to students that they have good physics ideas [56]—ideas that can serve as the basis for refinement and generalization *in physics contexts*. Even if their thinking changes and grows, we want them to recognize that physics understanding can start with their current thinking.

Preliminary observations suggest that the ACORN Physics Tutorial on electric circuits frequently elicits each of the resources we have identified in this paper. Further, we have video evidence that the ACORN Physics Tutorial on electric circuits frequently sparks and sustains students' sensemaking (as depicted in Ref. [57]) about circuits using these resources. We commonly observe that students ask rich questions, test them with simulated experiments [55], and seek causal or mechanistic explanations for what they observe. Common questions that students articulate as they work through the tutorial are as follows: Why does current going through a light bulb make it light? Why does the current out of the battery change when a bulb is added in parallel? Why does a lightbulb connected in series with a capacitor go out?

For example, a video-recorded group of four students working through the circuits ACORN Physics Tutorial were puzzled by the observation that a single bulb in series is equally as bright as two bulbs in parallel. One student asked: "So does that mean it's just the potential difference [across two bulbs in parallel] is the same? It's just pulling more current from the battery? Is that a thing that can happen?" This question centered on making sense of current as responsive—in their words, is a change in current a "thing that can happen?" This group of students went on to construct an accurate explanation for how more current can be "pulled" from the battery. After some discussion, one of the group members summarized: "[In] the parallel situation, the combined resistance with the two branches is going to be less than it is with A [a single bulb in series], so […] the resistance [is] less…that means current coming out of the battery is going to be higher." In this explanation, we see the students connect the "current is responsive" resource to the "connections" resource—describing how the parallel situation has less resistance than a single bulb in series—and the "resistance limits current" resource—describing how if the resistance is less, the current coming out of the battery will be higher.

The ACORN Physics Tutorial on circuits is just one example of how instruction might start with—and then build from—research on common conceptual resources. Our aim is to support not only systematic instructional materials development but also instructors' awareness and noticing of the generative ideas that students use as they reason about circuits. Along with other researchers [38], we hypothesize that an instructor's orientation to students' ideas matters for how instruction proceeds and for how students experience it. The resources that we characterize in this paper serve as a starting place for instructors who wish to plan ahead for the fruitful ideas that students in their classes may use, and there are many ways that instructors might choose to incorporate these ideas into their teaching.

ACKNOWLEDGMENTS

This work was supported in part by National Science Foundation Grants No. 1914603 and No. 1914572; all opinions are the authors' own. The authors are grateful for feedback at various stages from Ben Geller, Krishna Chawdary, Jayson Nissen, Vashti Sawtelle, Andrew Elby, Paula Heron, Rachel Scherr, Jonathan Corcoran, and Clausell Mathis. The authors would also like to acknowledge the land on which this paper was written: The University of Washington acknowledges the Coast Salish peoples of this land, the land which touches the shared waters of all tribes and bands within the Duwamish, Puyallup, Suquamish, Tulalip, and Muckleshoot nations [58]. Seattle Pacific University is named for a city that takes its name from Chief Si'ahl, the chief of the Duwamish people, whose territory and traditional land stretched across our campus. We respectfully acknowledge, with gratitude, that we work on the lands, both ceded and unceded, of the Coast Salish peoples, the land which touches the shared waters of all tribes and bands within the Suquamish, Tulalip, and Muckleshoot nations—people who are still here today [59].

APPENDIX

TABLE II. Information on universities and instructional context as reported by instructors.

University	Type	Location	Students	Key components of course
U1	Large public research university	Northwest United States	Intro, calc.-based physics class for science and engineering majors.	Lecture, lab, and tutorials [60]
U2	Large public research university	Central United States	Intro, calc.-based physics class for science and engineering majors.	Lecture (including clickers) and tutorials [60]
U3	Large public research university	Northeast United States	Intro, algebra-based physics class for biology, science, and healthcare majors.	Flipped class, SCALE-UP classroom environment [61]
U4	Mid-size private university	South United States	Intro, calc.-based physics class for science and engineering majors.	Lectures (including clickers) and tutorials [60]
U5	Small public university	Midwest United States	Intro, calc-based physics for science and engineering majors	Studio-style class with combined interactive lecture (including clickers), TIPERs [62] recitation, and lab
U6	Midsize private research university	Northeast United States	Intro, calc.-based physics class for science and engineering majors.	SCALE-UP classroom environment [61] Clicker questions
U7	Midsize public community college	Northeast United States	Intro, calc.-based physics class for science and engineering majors	Combined lecture or discussion section (including clickers), group problem solving and use of tutorials [60], and lab

TABLE III. The frequency of the "current is responsive" resource by question and university.

Current is responsive		
Question	Sample	Frequency (%)
Order-of-elements	U1	7
	U2	15
Modified-rank-the-bulbs	U1	40
	U3	11
Compare-bulbs-A, B, C	U4	24
	U5	0
	U6	17
	U3	3
Compare-bulbs-batteries-series	U1	37
	U6	33
	U5	17
	U3	6
Add-a-wire	U1	34
	U7	12
	U4	20

TABLE IV. The frequency of the "voltage drives current" resource by question and university.

Voltage drives current		
Question	Sample	Frequency (%)
Order-of-elements	U1	2
	U2	11
Modified-rank-the-bulbs	U1	0
	U3	0
Compare-bulbs-A, B, C	U4	0
	U5	0
	U6	0
	U3	0
Compare-bulbs-batteries-series	U1	37
	U6	33
	U5	17
	U3	3
Add-a-wire	U1	24
	U7	4
	U4	0

TABLE V. The frequency of the "resistance limits current" resource by question and university.

Resistance limits current		
Question	Sample	Frequency (%)
Order-of-elements	U1	2
	U2	9
Modified-rank-the-bulbs	U1	36
	U3	11
Compare-bulbs-A, B, C	U4	24
	U5	0
	U6	17
	U3	3
Compare-bulbs-batteries-series	U1	0
	U6	0
	U5	0
	U3	3
Add-a-wire	U1	21
	U7	8
	U4	17

TABLE VI. The frequency of the "connections" resource by question and university.

Connections resource		
Question	Sample	Frequency (%)
Order-of-elements	U1	51
	U2	28
Modified-rank-the-bulbs	U1	52
	U3	27
Compare-bulbs-A, B, C	U4	38
	U5	33
	U6	33
	U3	18
Compare-bulbs-batteries-series	U1	25
	U6	8
	U5	0
	U3	6
Add-a-wire	U1	38
	U7	46
	U4	37

[1] J. L. Docktor and J. P. Mestre, Synthesis of discipline-based education research in physics, Phys. Rev. ST Phys. Educ. Res. **10**, 020119 (2014).
[2] D. Hestenes, M. Wells, and G. Swackhamer, Force Concept Inventory, Phys. Teach. **30**, 141 (1992).
[3] L. C. McDermott, Millikan Lecture 1990: What we teach and what is learned—Closing the gap, Am. J. Phys. **59**, 301 (1991).
[4] L. C. McDermott, Oersted Medal Lecture 2001: "Physics Education Research—The Key to Student Learning", Am. J. Phys. **69**, 1127 (2001).
[5] L. Shulman, Knowledge and teaching: Foundations of the new reform, Harv. Educ. Rev. **57**, 1 (1987).
[6] D. Hammer, Student resources for learning introductory Physics, Am. J. Phys. **68**, S52 (2000).
[7] D. Hammer, A. Elby, R. E. Scherr, and E. F. Redish, Resources, framing, and transfer, in *Transfer of Learning from a Modern Multidisciplinary Perspective* (IAP, Greenwich, CT, 2005), pp. 89–119.
[8] E. Redish, A Theoretical Framework for Physics Education Research: Modeling Student Thinking (2004).
[9] A. A. diSessa, Toward an epistemology of physics, Cognit. Instr. **10**, 105 (1993).
[10] J. P. Smith, A. A. diSessa, and J. Roschelle, Misconceptions reconceived: A constructivist analysis of knowledge in transition, J. Learn. Sci. **3**, 115 (1993).
[11] J. Minstrell, *Student Thinking and Related Assessment: Creating a Facet-Based Learning Environment, in Grading the Nation's Report Card: Research from the Evaluation of NAEP*, edited by National Research Council (National Academies Press, Washington, DC, 2000).
[12] J. Minstrell, Facets of students' knowledge, and relevant instruction, in *Proceedings of an International Workshop held at the University of Bremen, 1991*, edited by R. Duit, F. Goldberg, and H. Niedderer (1992).
[13] R. Cohen, B. Eylon, and U. Ganiel, Potential difference and current in simple electric circuits: A study of students' concepts, Am. J. Phys. **51**, 407 (1983).
[14] J.-J. Dupin and S. Johsua, Conceptions of French pupils concerning electric circuits: Structure and evolution, J. Res. Sci. Teach. **24**, 791 (1987).
[15] P. V. Engelhardt and R. J. Beichner, Students' understanding of direct current resistive electrical circuits, Am. J. Phys. **72**, 98 (2003).
[16] T. V. Goris, Common misunderstandings of electricity: Analysis of interview responses of electrical engineering technology students, Int. J. Eng. Pedagogy **6**, 4 (2016).
[17] H. Küçüközer and S. Kocakülah, Secondary school students' misconceptions about simple electric circuits, Tufed-Tused **4**, 101 (2007), https://www.tused.org/index.php/tused/article/view/663/568.
[18] L. C. McDermott and P. S. Shaffer, Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding, Am. J. Phys. **60**, 994 (1992).
[19] A. Métioui, C. Brassard, J. Levasseur, and M. Lavoie, The persistence of students' unfounded beliefs about electrical

circuits: The case of Ohm's law, Int. J. Sci. Educ. **18**, 193 (1996).

[20] V. Picciarelli, M. D. Gennaro, R. Stella, and E. Conte, A study of university students' understanding of simple electric circuits Part 2: Batteries, Ohm's law, power dissipated, resistors in parallel, Eur. J. Eng. Educ. **16**, 57 (1991).

[21] D. M. Shipstone, C. v Rhöneck, W. Jung, C. Kärrqvist, J.-J. Dupin, S. Johsua, and P. Licht, A study of students' understanding of electricity in five European countries, Int. J. Sci. Educ. **10**, 303 (1988).

[22] C.-H. Tsai, H.-Y. Chen, C.-Y. Chou, and K.-D. Lain, Current as the key concept of Taiwanese students' understandings of electric circuits, Int. J. Sci. Educ. **29**, 483 (2007).

[23] B. W. Harrer, V. J. Flood, and M. C. Wittmann, Productive resources in students' ideas about energy: An alternative analysis of Watts' original interview transcripts, Phys. Rev. ST Phys. Educ. Res. **9**, 023101 (2013).

[24] D. Hammer and E. van Zee, *Seeing the Science in Children's Thinking: Case Studies of Student Inquiry in Physical Science*, 1st ed. (Heinemann, Portsmouth, NH, 2006).

[25] D. Hammer, Discovery learning and discovery teaching, Cognit. Instr. **15**, 485 (1997).

[26] D. Hammer, Student inquiry in a physics class discussion, Cognit. Instr. **13**, 401 (1995).

[27] P. M. Heller and F. N. Finley, Variable uses of alternative conceptions: A case study in current electricity, J. Res. Sci. Teach. **29**, 259 (1992).

[28] A. Leniz, K. Zuza, and J. Guisasola, Difficulties understanding the explicative model of simple DC circuits in introductory physics courses, presented at PER Conf. 2014, Minneapolis, MN, 10.1119/perc.2014.pr.034.

[29] P. Licht and G. D. Thijs, Method to trace coherence and persistence of preconceptions, Int. J. Sci. Educ. **12**, 403 (1990).

[30] D. Smith and P. van Kampen, Teaching electric circuits with multiple batteries: A qualitative approach, Phys. Rev. ST Phys. Educ. Res. **7**, 020115 (2011).

[31] S. Bull, T. J. Jackson, and M. J. Lancaster, Students' interest in their misconceptions in first-year electrical circuits and mathematics courses, Int. J. Electr. Eng. Educ. **47**, 307 (2010).

[32] B.-S. Eylon and U. Ganiel, Macro-micro relationships: The missing link between electrostatics and electrodynamics in students' reasoning, Int. J. Sci. Educ. **12**, 79 (1990).

[33] D. M. Shipstone, A study of children's understanding of electricity in simple DC circuits, Eur. J. Sci. Educ. **6**, 185 (1984).

[34] D. M. Shipstone, *Electricity in Simple Circuits, in Children's Ideas in Science* (Milton Keynes Open University Press, Philadelphia, 1985).

[35] D. E. Brown and D. Hammer, Conceptual change in physics, in *International Handbook of Research on Conceptual Change* (Routledge, London, 2008).

[36] R. E. Scherr, Modeling student thinking: An example from special relativity, Am. J. Phys. **75**, 272 (2007).

[37] B. W. Harrer, Identifying productive resources in secondary school students' discourse about energy, The University of Maine, 2013, https://digitalcommons.library.umaine.edu/etd/2065/.

[38] D. Hammer, Misconceptions or p-prims: How may alternative perspectives of cognitive structure influence instructional perceptions and intentions, J. Learn. Sci. **5**, 97 (1996).

[39] A. Elby, R. E. Scherr, T. McCaskey, R. Hodges, E. F. Redish, D. Hammer, and T. Bing, Maryland Open-Source Tutorials in Physics Sense-Making, Part 1 (2007).

[40] T. D. Cook, Randomized experiments in educational policy research: A critical examination of the reasons the educational evaluation community has offered for not doing them, Educ. Eval. Policy Anal. **24**, 175 (2002).

[41] L. M. Goodhew, A. D. Robertson, P. R. L. Heron, and R. E. Scherr, Students' context-sensitive use of conceptual resources: A pattern across different styles of question about mechanical waves, Phys. Rev. Phys. Educ. Res. **17**, 010137 (2021).

[42] M. B. Miles and A. M. Huberman, *Qualitative Data Analysis: An Expanded Sourcebook*, 2nd ed. (Sage Publications, Inc, Thousand Oaks, CA, 1994).

[43] S. Kanim and X. C. Cid, Demographics of physics education research, Phys. Rev. Phys. Educ. Res. **16**, 020106 (2020).

[44] J. C. Ng, S. S. Lee, and Y. K. Pak, Chapter 4 Contesting the model minority and perpetual foreigner stereotypes: A critical review of literature on Asian Americans in education, Rev. Res. Educ. **31**, 95 (2007).

[45] National Center for Education Statistics, Undergraduate enrollment, https://nces.ed.gov/programs/coe/indicator/cha.

[46] L. M. Goodhew, A. D. Robertson, P. R. L. Heron, and R. E. Scherr, Student conceptual resources for understanding mechanical wave propagation, Phys. Rev. Phys. Educ. Res. **15**, 020127 (2019).

[47] A. D. Robertson, L. M. Goodhew, R. E. Scherr, and P. R. L. Heron, University student conceptual resources for understanding forces, Phys. Rev. Phys. Educ. Res. **17**, 010121 (2021).

[48] F. Marton, Phenomenography—A research approach to investigating different understandings of reality, J. Thought **21**, 28 (1986), https://www.jstor.org/stable/42589189.

[49] K. H. Krippendorff, *Content Analysis—3rd edition: An Introduction to Its Methodology* (SAGE Publications, Inc, Thousand Oaks, CA, 2013).

[50] A. D. Robertson, L. M. Goodhew, L. C. Bauman, B. Hansen, and A. T. Alesandrini, Identifying student conceptual resources for understanding physics: A practical guide for researchers, Phys. Rev. Phys. Educ. Res. **19**, 020138 (2023).

[51] M. Banerjee, M. Capozzoli, L. McSweeney, and D. Sinha, Beyond Kappa: A review of interrater agreement measures, Can. J. Stat. Rev. Can. Stat. **27**, 3 (1999).

[52] L. C. McDermott and Physics Education Group, *Physics by Inquiry: An Introduction to Physics and the Physical Sciences* (John Wiley & Sons, New York, 1995), Vol. 2.

[53] R. Mondesir and A. D. Robertson, Toward characterizing the demographics of Introductory Physics Courses, presented at PER Conf. 2020, Virtual conference, 10.1119/perc.2020.pr.Mondesir.

[54] A. D. Robertson, L. M. Goodhew, L. C. Bauman, and P. R. L. Heron, ACORN Physics Tutorials, https://www.physport.org/curricula/ACORN/.

[55] A. Rouinfar, S. Reid, D. Barnett, W. Adams, M. Dubson, A. Paul, K. Perkins, C. Wieman, M. Hermsmeyer, and C. McCutchan, Circuit Construction Kit: DC, https://phet.colorado.edu/en/simulations/circuit-construction-kit-dc/about.

[56] E. Duckworth, *The Having of Wonderful Ideas" and Other Essays on Teaching and Learning*, 3rd ed. (Teachers College Press, New York, 2006).

[57] T. Ole, B. Odden, and R. S. Russ, Sensemaking epistemic game: A model of student sensemaking processes in introductory physics, Phys. Rev. Phys. Educ. Res. **14**, 020122 (2018).

[58] Native Life & Tribal Relations, https://www.washington.edu/diversity/tribal-relations/.

[59] Acknowledging the Coast Salish Peoples, https://spu.edu/diversity/spu-diversity-commitment/oiex-land-page.

[60] L. C. McDermott and P. S. Shaffer, *Tutorials in Introductory Physics* (Pearson College Div, Upper Saddle River, NJ, 2001).

[61] R. J. Beichner, SCALE-UP, https://www.physport.org/methods/method.cfm?G=SCALE_UP.

[62] C. Hieggelke, D. Maloney, S. Kanim, and T. O'Kuma, Tasks inspired by physics education research, https://www.physport.org/methods/method.cfm?G=TIPER.