COORDINATING PRACTICES: IN-SERVICE SECONDARY TEACHERS' USE OF 5-PRACTICES TO SUPPORT MATHEMATICS DISCUSSIONS

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We present three cases of how in-service secondary teachers took up the 5-Practices instructional framework to support mathematics discussions in their classrooms. The data comes from the teachers first implementation of researcher designed and teacher modified mini-units in their classrooms that were part of a 3-year design experiment. The cases illuminate common yet subtly different affordances and challenges that the in-service teachers experienced in using the 5-Practices. They also highlight how the research team adjusted the professional support they provided to the teachers after the first implementation of the mini-units. One outcome of this adjustment was the development of a multi-tiered framework that relates teaching moves, practices, instruction, and instructional routines. The findings and framework contribute to the body of research on how in-service teachers can learn to support mathematics discussions.

Keywords: Core practices, teacher learning, mathematics discussion, design experiments

Mathematics educators have characterized rich mathematical discussions as ones that are centered on students' mathematical reasoning and are aimed at accomplishing specific instructional goals (Jacobs & Spangler, 2017). As such, mathematical discussions have been found to be a vehicle through which students have opportunities to learn substantive mathematics (Boaler & Staples, 2008). Opportunities for learning arise as students clarify (Goos, 2004), refine (Richland et al., 2019), expand (Webb et al., 2014), generalize (Land et al., 2014), and justify (Brodie, 2010) their mathematical reasoning through sharing it with their peers and by engaging with their peers' reasoning. Facilitating productive mathematics discussions relies on complex teaching capacities including making quick decisions, eliciting students' thinking, being responsive to students' contributions, and managing cognitive demand. Given these complexities, researchers have found that teachers often need support to effectively learn to facilitate productive discussions (Boston & Smith, 2009) where this support often occurs through decompositions, representations, and approximations of practice (e.g., Staples & Truxaw, 2010).

We use this paper to report on a cross-case comparison of three experienced secondary teachers who were learning to adapt and incorporate the *5 Practices* (5Ps) (Smith, Steele & Sherin, 2020) into their teaching. The teachers were part of a 3-year design experiment whose aim was to study how secondary in-service teachers facilitated students' *mathematical generalizing*. We introduced the teachers to the 5Ps framework for two reasons: (a) we considered ourselves more likely to see high quality and varied instances of how teachers' supported student generalization if the teachers were also supporting student discussion; and (b) we wanted teachers to have a common instructional framework to support cross-classroom conversations about instruction. The data we report on in this paper is from the first time the teachers implemented researcher-designed and teacher modified mini-units in their classroom.

We respond to the following research question: What affordances and challenges did the teachers experience in their re-composition of the 5Ps over the course of implementing their mini-unit?

Literature Review

Many early studies that investigated how teachers facilitated productive mathematical discussions occurred in classrooms with teachers who had considerable content and pedagogical expertise: often the teacher was a researcher (e.g., Ball, 1993, Heaton, 2000; Lampert, 1990). These studies yielded substantial information about teaching practices that support mathematics discussions. Subsequently, mathematics educators transformed this information into practitioner friendly frameworks that could be used to support both pre- and in-service teachers (PSTs and ISTs, respectively) to support mathematical discussions (e.g., Chappin, et al., 2003/2013; Stein, et al., 2008). Researchers have used these frameworks with PSTs in mathematics methods courses (e.g., Ghousseini & Herbst, 2016; Tyminski, et al., 2014) and with ISTs in professional development settings (e.g., Reinsburrow, et al., 2022) to study the PSTs and ISTs learning of discussion-based practices. However, Ghousseini (2015) and Pang (2016) both identified a dearth of research that examines how PSTs or ISTs learn to use discussion-based frameworks *in actual classrooms* (as opposed to in methods courses or professional development settings).

Researchers (e.g., Bağdat & Yanik, 2023; Heyd-Metzuyanim et al., 2019; Kooloos, et al., 2023; Martins et al., 2023) have begun to respond to this lack of research. Within this work, they have reported that practices that occur prior to teaching (e.g., setting goals, identifying tasks, and anticipating) are easier for teachers to learn than those that occur during teaching (e.g., monitoring, selecting, sequencing, and connecting) (Pang). We use this study to contribute to this growing body of research by identifying how teachers adapt and incorporate discussion-oriented practices, specifically the 5Ps, into classroom instruction. We were particularly interested in how the teachers transitioned from using decompositions, representations, and approximations of practice in professional development to implementation of these practices in live instruction.

Analytic Framework

Jacobs and Spangler (2017) define teaching moves as “actions that teachers take that observers can see or hear, such as asking a question, providing a representation, or modifying a task” (p. 778). They differentiate teaching moves from goals, which they define as “the intentions teachers have….(which) typically must be inferred by researchers because they are not usually stated explicitly” (p. 778). Jacobs and Spangler acknowledge that moves and goals take place at different grain sizes. However, they do not introduce a language to differentiate among the different grain sizes. To capture these differences in grain size, we introduce a four-tiered nested framework that preserves Jacobs’s and Spangler’s distinction between observable actions and goals a teacher has for these actions. Moving from smallest to largest grain size, we use the term *teaching move* to mean actions that occur during moments of interaction with students. *Teaching practices* (Smith & Stein, 2017) occur over a longer timeframe within a lesson where multiple teaching moves are embedded in each teaching practice. Teaching practices, then, function together to comprise *instruction* where we use the term instruction to refer to teaching that occurs in a single lesson. We use the term *instructional routines* for patterns in a teacher’s instruction that can be discerned over multiple days of instruction. At each grain size, it is possible to characterize both a teacher’s observable actions and their goals.

In outlining the 5Ps, Smith, et al. (2020) do not make the same distinctions we do in grain size, but it is possible to interpret the 5Ps relative to these distinctions, particularly the distinction between teaching moves and practices. In Table 1 we define Smith, et al.’s practices that occur while teaching, Practices 2-5—monitoring, selecting, sequencing, and connecting—and give examples of observable actions and goals that are at the practice and move grain size. In Table 1, we omit Practice 0, setting goals/selecting tasks, and Practice 1, anticipating, because they occur prior to teaching, and our primary interest in this paper is what happened while teaching.

Table 1: Monitoring, Selecting, Sequencing, and Connecting Practices (Smith et al., 2020)

5Ps Defined	Actions Related to Teaching Practice	Goal(s) of the Teaching Practice	Actions Related to Teaching Moves	Goals of the Teacher Moves
Monitoring: Attending to student thinking while students work on a problem	• Circulates among groups of students, revisiting groups when appropriate • Asks assessing and advancing questions	• Track student thinking • Assess student thinking • Advance student thinking (p. 86)	• Asks an assessing or an advancing question • Uses a talk move	• Understand student thinking • Move a student toward a learning goal
Selecting & Sequencing: Choosing what student work to discuss and organizing that work in a specific order	• Records the range of strategies from which to choose (selecting) • States the order of student presentations (sequencing)	• Ensure student work to be shared connects to all learning goals • Establish a coherent storyline for the work presented (p. 122)	• Asks a student (privately) if they will share their work • Calls on a specific student to share their work first	• Ensure a student is willing to present • Indicate to an individual student when to share
Connecting: Using student work to make connections to learning goals or connections among different pieces of student work	• Asks questions to all presenters to highlight connections to learning goals • Records student observations about similarities and differences across all student work	• Connects student work to the full range of learning goals for the lesson • Connects the *set* of selected student work to each other (p. 172)	• Asks a question to highlight how a particular piece of student work connects to a learning goal(s) • Has students turn-and-talk about how a new piece of work is related to their own	• Makes connections between an individual presenter's work and learning goals • Makes connections between two specific pieces of student work

The four-tiered framework was an *outcome* of the first year of our design work. We introduce it here because it supports what follows. We see the distinctions in grain size as important to framing the claims a researcher is making about a teacher's teaching. That is, it is different to make claims about a teacher experiencing success at the smallest grain size, teaching moves, than it is to make claims about a teacher experiencing the same success at the largest grain size, instructional routines. It is different because the grain size of an action impacts the ease with which a teacher will successfully integrate that action into their instruction. For example, it is easier for a teacher to introduce smaller grain size teaching moves like wait time, re-voicing, asking an assessing question, or inviting students to participate than it is to adopt larger grain size instructional routines like consistently using the 5Ps over the course of a mini-unit. Our aim in this study is to describe instructional routines that we could discern over multiple lessons taught by the ISTs; in particular, we investigate the way that the ISTs fit together practices, from the 5Ps, in instruction, and identify patterns (i.e., instructional routines) related to how they did so. Although important, we have a smaller focus on specific teaching moves and practices.

Methods and Methodology

Design experiment research involves researchers designing an intervention, testing that intervention, and then refining the intervention during subsequent iterations (Cobb et al., 2003). As part of this process, researchers identify conjectures they have that guide the design of the intervention, where one result of a study involves documenting how they modified their conjectures for future iterations of the intervention. We focus on one conjecture we made related to our work with the teachers relative to the 5Ps. *Conjecture:* Decompositions, representations, and approximations of *individual teaching practices* and *moves* within practice was sufficient support for ISTs to recompose these practices in instruction in ways that would support them to develop reliable new instructional routines.

During the summer, prior to implementing the mini-units in their classrooms, all three teachers participated in eleven, 3-hour professional collaboration sessions. The first and third authors designed these sessions to focus on four themes: (a) the mathematical content of the mini-units; (b) student reasoning related to the mathematical content, which included video cases (Burch, et al., 2021); (c) instructional planning for the mini-units using the 5Ps as a guide for this planning (Smith, Steel, Sherin, 2020); and (d) a teacher-appropriate framework for supporting generalization in the classroom (Driscoll, 1999). The mini-units were initially developed by the third and first author (Burch & Tillema, unpublished) and were subsequently modified by the teachers as part of the process of planning for the implementation. Each mini-unit lasted 3-7 days, depending on the length of each teachers' class period (i.e., 45-, 50-, and 70-minutes). All lessons in the mini-unit were videotaped using three cameras: one captured the whole classroom, one captured the teachers' interactions with students, and one focused on a small group of students as they participated in the lessons. The three participating teachers—Celine, Felix, and Hazel—had 30, 21, and 6 years of teaching experience, respectively. We use Table 2 to briefly characterize classroom structures and opportunities for discussion in each teachers' classroom. The observations are based on field notes taken during five visits to each teacher's classroom and an interview about their instruction, which occurred in the year prior to teaching the mini-units. None of the teachers used student driven discussion as the primary organizational tool for instruction in their classrooms, but all incorporated, to varying degrees, elements of discussion.

Table 2: Brief Characterization of Teacher' Instruction Prior to the Project

Celine	Celine relied largely on problems from traditional textbooks in which more open-ended application problems occurred after problems involving a particular skill. Celine's normal instructional routine was to have students begin class in small groups to discuss the previous evening's homework. She, then, introduced new content through a teacher led problem solving session, which she considered a guided-discovery approach. Celine's guided discovery consisted of her publicly solving problems where she directed the solution of the problem, but students were expected to contribute key pieces to the solution of a problem. Celine had carefully identified the key pieces students were expected to contribute based on what they had already worked on. Celine, then, offered students time to solve several similar problems in small groups where she had an array of mechanisms in place to support student to student interactions. She assigned 3-5 homework problems at the end of class to work on outside of class.
Felix	Felix relied largely on problems from traditional textbooks. He began class with two students each presenting a homework problem where students explained their solution to the class. During this time, other students in the class asked the presenters questions. Felix, then, used the student presentations to highlight key ideas that he anticipated other students might have struggled to understand. After student presentations, Felix gave a lecture on the topic for the day, with some students taking notes and others listening to the information. Felix, then, assigned 8-10 homework problems and gave students time to work.
Hazel	Hazel often started her class with an open-ended problem that students worked on individually, in small groups, or as a whole class. She used the problem to hook students and highlight key ideas she intended for them to work on in class that day. Once the class discussed this problem, they typically worked either individually or in small groups on more common textbook problems. The outcome of this work was for students to present their solutions to other students either in small groups or whole class. Hazel, then, assigned students 3-5 homework problems at the end of class to work on outside of class.

For analysis, we mixed the whole classroom video and teacher video into a single video file. One mathematics education faculty member and six graduate students coded 45-minute segments of Celine's mini-unit using the 5Ps framework. After coding a 45-minute segment, the team met to discuss and refine codes where this part of analysis focused on what observable actions corresponded to codes for each practice. The research team continued this process until coding for Celine's entire mini-unit was complete; at this point code definitions were relatively stable. Next, the team coded one lesson from Felix and one from Hazel to make additional minor adjustments in the code book based on differences across the three teachers' instruction. The result of this process was a code book with stable code definition for 10 codes: launch; monitoring; assessing questions; advancing questions; selecting: evidence of teacher choosing student’s work; selecting: evidence of what student work was presented; sequencing; connecting to learning goals; connecting student work; and student work time. The research team, then, broke into two subgroups, and used the, now stable, code book to code the remaining lessons from Felix and Hazel’s mini-units. This coding occurred similarly to what is described above. To respond to our research question, we used descriptive statistics to capture the percentage of time each teacher spent on each practice, which supported our qualitative interpretations of how the teachers recomposed the 5Ps in instruction and what patterns emerged as instructional routines.

Results

In Table 3, we identify the percentage of time and coding frequency for each code. We use this information to highlight salient features in each teacher’s re-composition of the practices in instruction to characterize their instructional routines. It is important to note that none of the teachers wanted to use a monitoring chart (cf. Smith et al., 2020) the first time they implemented their mini-units. Their concerns were rooted in perceived trade-offs between a monitoring chart’s helpfulness to organize their thinking and distraction from staying present with their students.

Table 3: Descriptive Statistics Related to Each Practice[1]

	Felix			Celine			Hazel		
# of Lessons	3			7			5		
Total Time	3:45:59			5:44:16			3:53:15		
	Total Time (hr:min:sec)	**Coverage (%)**	**Code Fre-quency**	**Total Time (hr:min:sec)**	**Coverage (%)**	**Code Fre-quency**	**Total Time (hr:min:sec)**	**Coverage (%)**	**Code Fre quency**
Launch	57:00	25.22%	13	57:26	16.69%	33	47:59	20.57%	17
Monitoring	1:08:45	30.43%	16	1:57:57	34.27%	21	1:56:23	49.90%	20
Assessing Questions	7:02	3.11%	11	10:52	3.16%	20	34:31	14.80%	75
Advancing Questions	6:49	3.02%	8	22:03	6.41%	43	47:13	20.25%	59
Selecting (Choosing)	11:38	5.15%	11	9:37	2.79%	38	3:44	1.60%	16
Selecting (Presenting)	31:28	13.92%	11	34:47	10.11%	39	12:51	5.51%	19
Sequencing	25:33	11.31%	2	49:48	14.47%	9	16:06	6.91%	7
Connecting Learning goal	57:31	25.46%	8	1:41:10	29.39%	46	35:32	15.23%	21
Connecting Student work	31:11	13.81%	9	3:01	0.88%	3	10:11	4.37%	10
Student work time	6:54	3.06%	4	22:12	6.45%	10	0	0.00%	0

Felix spent 30.43% of the total instructional time monitoring students, while they worked on problems. Of that time, he spent the lowest percentage of time asking assessing or advancing questions (i.e., 10.18% and 9.80%, respectively, for a total of 19.98% of his monitoring time[2]). Instead, Felix tended to listen to and observe student-to-student conversations while they worked in small groups. For 15.35% of the time he was monitoring, we double-coded selecting (choosing)—a code we used when there was observable evidence that a teacher was choosing a particular student’s work to share later in the lesson. We infer, then, that, while he was listening to and observing student conversation during monitoring, he was also focused on determining what work he would *select* to have students share with the class. Moreover, Felix did not have any sequencing codes that occurred while he was monitoring. This indicates there was no observable evidence that he was considering how to sequence student work while monitoring. We infer from this combination of codes, and our qualitative observations, that Felix was challenged to coordinate asking students assessing and advancing questions (relatively low percentage of his monitoring time) while also aiming to determine what student work to select

[1] Percentages exceed 100% because of overlapping codes (e.g., monitoring and assessing questions).

[2] These percentages differ from those in the table because they represent the percent of time he was asking each kind of question relative to the total time he was monitoring rather than relative to the total time of instruction.

(relatively high percentage of his monitoring time) and sequence that work (none of his monitoring time). Our inference is that he was heavily focused on what work he would select while he was monitoring over, for example, assessing differences in student thinking.

Among the teachers, Felix had the highest percentage of time coded for selecting (presenting) (13.92%) and the lowest number of code instances (11 coded instances). We used this code when students presented their own work to the class. This combination of codes meant that Felix had *fewer, but substantially longer periods of time* during which students presented their work to the whole class than the other teachers. We attribute the length of student presentations to two factors: Felix spent minimal time asking assessing and advancing questions, which meant that sometimes significant mathematics surfaced for the first time during these presentations; and he, among the teachers, allowed for the most open ended whole class discussion of student ideas.

Felix managed the sequencing practice by having multiple students put their work on the white board at the same time. Doing so meant that he had very few instances of the code sequencing, in part, because all student work was displayed simultaneously with students often comparing multiple pieces of student work to each other at the same time. One affordance of this instructional decision was that Felix had ample opportunity to engage in the practice of connecting student work (13.81% of his total instructional time). This code was double coded with selecting (presenting) for 63.38% of the total selecting (presenting) time. Thus, Felix often actively asked questions of students as they presented their work—questions that focused on making connections to other students' work. We also coded a substantial portion of Felix's total instructional time as connecting to learning goals (25.46%); however, this code was only double coded for 4.6% of the total time coded selecting (presenting). This indicated that Felix often made connections to learning goals after students presented their work either by asking further questions of them or by making his own explicit statements of connection.

Celine spent 34.27% of her instructional time monitoring student work. She spent a relatively low percentage of her *monitoring time* asking assessing questions (9.18%) with a larger percentage of that time spent asking advancing questions (18.64%) for a total of 27.82% of her monitoring time. Celine, like Felix, spent much of her time monitoring by listening intently to small group conversations and observing the work that students produced during this time. One reason Celine had a lower percentage of time coded for assessing questions than advancing questions was she often used her assessing listening as a basis to ask advancing questions.

Celine's data also indicates that she experienced a challenge in coordinating asking assessing and advancing questions with engaging in initial phases of selecting and sequencing while monitoring. However, this challenge expressed itself differently in her re-composition of the practices in instruction than it did in Felix's instruction. That is, in contrast to Felix, only 1.6% of her total time monitoring was double coded with selecting (choosing). Her monitoring, then, included little observable evidence that she was considering what student work she would select.

Celine had a lower percentage of total instructional time coded for selecting (presenting) (10.11%) as compared to Felix (13.92%), but a high number of instances of the selecting (presenting) code (39 coded instances). This set of code combinations indicates that she had *frequent but short times* during which *multiple* students had the opportunity to present their work, and they presented sequentially. She did have a substantial percentage of her total instructional time coded as connecting to learning goals (29.39%). However, the code connecting to learning goals was double coded only 1.4% of the total time the selecting (presenting) code was used. This indicates that Celine tended to have students present their work, and then once they had presented it, she made connections to learning goals by either asking the class additional questions or by making her own explicit statements of connection to the learning goals. This sequential code structure indicates that Celine often prepared questions to ask the class as students presented their work but did not integrate this questioning into student presentations. Another consequence of Celine's students presenting their work one-by-one was that, in many instances, the record of student work was gone after it was presented. As such, it was challenging for Celine to make connections across student work. This is supported by only 0.88% of her total instructional time being coded as connecting student work; moreover, this code was never double coded with the selecting (presenting) code.

Hazel spent 49.90% of her total instructional time engaged in the practice of monitoring, a substantially higher percentage than either Felix or Celine. Hazel also spent 70.28% of the time she was coded as monitoring asking either assessing or advancing questions (29.66% and 40.62%, respectively)—also substantially higher than Felix or Celine. Among the three teachers, Hazel's monitoring practice provided the most concrete, observable evidence that she was using her time monitoring to make sense of details about her students' thinking and, therefore, would be well-positioned to both choose student work to share with the class and to consider a sequence for this work. However, only 0.01% of her monitoring time was double coded with selecting (choosing), meaning there was little observable evidence she was engaged in the initial processes of selecting student work while monitoring and no evidence she was considering sequencing it. This code structure again indicates that Hazel was challenged to coordinate monitoring with the initial phases of selecting and sequencing student work.

We infer a challenge for Hazel was balancing the time she spent monitoring with foreseeing the amount of time she would need to effectively engage her class in whole-class discussion about their work. Of the three teachers, she had the lowest percentage of her instructional time coded as selecting (presenting) (5.51%), as sequencing student work (6.91%), and connecting to learning goals (15.23%). These percentages support our observation that she often did not have sufficient time at the end of her lesson to connect to learning goals even though she did engage in this practice at the end of each lesson. The combination of percent of instructional time with frequency for the selecting (presenting) code indicates that Hazel was in between Celine's frequent, short student presentations, and Felix's less frequent, longer student presentations.

The percentage of instructional time coded selecting (presenting) that was double coded as connecting to learning goals was 59.33%. This indicated that, when students were presenting their work, Hazel was often actively questioning them in ways that supported connecting to learning goals, a phenomenon we attribute to her careful use of assessing and advancing questions while monitoring. Overall, 4.37% of Hazel's total instructional time was coded as connecting student work—again between Felix (Felix had a percentage about 3 times higher) and Celine (Celine had a percentage about 1/5 as much). Hazel, like Celine, frequently had multiple students present their work one-by-one often without having a way to simultaneously display multiple pieces of student work. This organization for presenting student work meant that, while students were presenting, Hazel focused on connecting to learning goals rather than connecting student work, which did not occur as a double code with selecting (presenting).

Discussion

One possible way to read the data is that each of the three teachers, in one way or another, was relatively far away from a high-level implementation of the 5Ps. We caution against this interpretation; we were specifically interested in documenting the affordances and challenges that experienced ISTs faced when coordinating the practices together in *instruction* for the first time, and how to support them in the emergence of new *instructional routines*. With this observation, we return to the *conjecture* that guided our design: Decompositions, representations, and approximations of *individual* practices and *moves* within practice was sufficient support for in-service teachers to recompose these practices in instruction in ways that would support them to develop reliable new instructional routines. During the professional collaboration sessions, the ISTs were all able to engage with the practices individually, demonstrating, for example, what we considered to be high level approximations of each individual practice. However, in their classroom teaching, a substantial challenge they faced was how to coordinate the practices with each other in instruction to produce new reliable instructional routines. Moreover, that was how they judged their success; they judged their success, at least initially, in terms of how the practices fit together within a single lesson, and how multiple days of instruction produced new reliable instructional routines—criteria that were neither explicit for them, nor to us, until they implemented the mini-units. Given these observations, we refine our conjecture: the ISTs needed support, in decompositions, representations, and approximations, that focused more explicitly on coordinating practices with each other to produce reliable instruction and instructional routines.

The three cases offer insight into what this support could look like. That is, one common challenge across all three teachers was to coordinate monitoring with the early phases of selecting and sequencing. This challenge may have been due to the teachers' choice not to use a monitoring chart, however, we do not attribute this challenge only to this decision. Moreover, there were subtle differences in their experience of this challenge, and thus differences in the support that could address it. Felix, for example, while monitoring, was consumed with what work to select, and as such asked a relatively low number of assessing and advancing questions. His listening, while monitoring, was often focused on what work he would choose rather than using an assessing, and then advancing question cycle to help him determine what students were thinking and then to move that thinking forward. On the other hand, Hazel focused extensively on asking assessing and advancing questions, while monitoring. Doing so gave her the most detailed information about her students' thinking and thus prepared her to ask students questions to make explicit connections to learning goals. However, there were very few observable teaching actions focused on preparing to select student work, which was related to her inefficiency in transitioning from monitoring to selecting and sequencing student work. These different challenges call for differences in support each teacher needed.

We close by identifying one contribution of this study. Research reports on teachers' use of the 5Ps often focus on characterizations of an individual practice (e.g., Dunning, 2022; Reinsburrow, et al., 2022; Tyminski, et al., 2014) even in reports where multiple of the practices are considered (e.g., Bağdat & Yanik, 2023). These accounts offer important details about a specific practice, including the distinct goals and uses that teachers have for the practice. We think another important point of focus is on how teachers learn to coordinate the practices together (e.g., Pang, 2016), and how this coordination evolves over time into new instructional routines. Our assertion is supported by Felix, Celine, and Hazel's initial judgements of the success of a given lesson (i.e., their instruction) as based—*not* on their use of individual practices, but rather—on how the practices fit together for them within the lesson and across the mini-unit. Given that experienced ISTs' instruction is rooted in established instructional routines, working with them to experience what they deem to be successful instruction is important as they determine for themselves whether to adopt new practices that alter established routines.

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