3D Multi-plane Multi-resolution Shear Wave Elastography

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***Abstract*— We utilize Full Waveform Inversion (FWI) framework to convert shear wave elastography (SWE) measurements on multiple planes to a single three-dimensional (3D) image of elasticity using gradient optimization to adjust the elasticity map until the mismatch between simulated and measured particle velocities is minimized. Several ideas are brought together to ensure the robustness and effectiveness of the proposed FWI: correlation-based matching between measurements and simulation, high-fidelity finite element simulation of 3D shear waves in incompressible elastic media, multi-resolution parametrization to help with convergence, multi-frequency continuation for finer resolution imaging, and multi-push illumination. With the help of *in silico* and phantom validation studies, the algorithm is shown to be effective in providing images outside the measurement plane, including reconstructing full 3D images from 2D SWE measurements.**

Keywords—full waveform inversion, multi-frequency imaging

I. Introduction

In shear wave elastography (SWE), acoustic radiation force (ARF) excites a location of the soft tissue, causing waves to propagate and scatter in three-dimensions (3D). However, particle velocities are measured only in a two-dimensional (2D) plane with no information in the elevational direction. Due to such 2D measurements, most existing SWE algorithms focus on providing 2D images within the measurement plane. Our objective, however, is to obtain a fully 3D image of tissue elasticity including in the elevational direction.

II. Methods

We utilize Full Waveform Inversion (FWI) framework to convert SWE measurements on multiple planes to a single 3D image of elasticity using gradient optimization to adjust the elasticity map until the mismatch between simulated and measured particle velocities is minimized. As typical, FWI is formulated as a partial differential equation (PDE) constrained optimization problem where the objective is to minimize the misfit between measured and simulated particle velocities, where the simulated particle velocities satisfy the underlying PDE. Optimization is performed using gradient methods based on adjoint formulation. While the abstract formulation of FWI is fairly standard, care must be exercised to ensure convergence to the correct image. Several ideas are brought together for this purpose: (a) correlation-based matching between measured and simulated particle velocities; (b) high-fidelity finite element simulation of 3D shear waves; (c) multi-resolution parametrization of elasticity maps; (d) refinement of multi-resolution imaging through frequency continuation; (e) multi-plane illumination to increase sensitivity. In this short summary paper, we outline our approach in each of these dimensions; details can be found in [1] as well as upcoming publications.

A. Correlation-based Matching

Typical objective functions used in FWI are based on the least-squares norm of the misfit between the measured and simulated values. In this work, we utilized the correlation objective function, which is shown to have smoother optimization terrain leading to faster convergence.

B. High-Fidelity Finite Element Simulations

While most existing SWE imaging approaches idealize the wave propagation using a scalar wave equation in 2D, a more accurate version would be the 3D elastodynamics equation. We solve this equation using finite elements implemented in MOOSE [2], with appropriate incompressibility constraints and absorbing boundary conditions.

C. Multi-resolution Prametrization

The main limitation of gradient-based optimization methods, when applied to ill-posed inverse problem like FWI, is their tendency to converge to local minima, often far from the true solution. This is particularly true when the number of parameters is large. This problem can be tackled, e.g., by using explicit regularization functions or by gradual expansion of parametrization. We employ the latter approach, where the inversion is first performed using a coarse mesh. The parameter space is then expanded by refining the underlying (parameter) mesh and the process is repeated. While the refinement details can be problem dependent, we were able to start with a coarse mesh, e.g. 3-4 points in each spatial

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direction and reduce the spacing to half for each refinement step (see the illustrative example related to Fig. 1).

D. Muli-frequency Continuation

While performing parameter mesh refinement, it is sometimes beneficial to modify the objective function to gradually expand the frequency window over which the measured and simulated data are matched. The details of the frequency window should be determined based on the expected modulus range and the frequency range of reliable data acquisition. Such modification of the objective function gradually increases the sensitivity as the parameter mesh is refined – a strategy that has proven to be beneficial in our investigations as well.

E. Multi-plane Illumination

A fundamental requirement for the success of any elasticity imaging algorithm is that the measurements must be sensitive to material property perturbation. While this is generally the case for in-plane SWE imaging, for out-of-plane imaging, the SWE measurements may not be sensitive to the material properties farther from the measurement plane, especially in the presence of attenuation. To circumvent this, we propose to illuminate and measure waves in multiple SWE planes, which are then combined in the FWI framework in a straightforward manner.

The FWI methodology developed here is implemented into the MOOSE optimization module [3], a recently developed framework for PDE constrained optimization.

III. Results

In silico validation is performed by imaging the shear modulus map (G) of 2 cm x 2 cm x 3 cm region of a liver-mimicking material with diffuse disease (Fig. 1). FWI is initially conducted on 4 x 4 x 4 parameter grid (*6.7 mm* resolution in *x* and *z* directions and *10 mm* in *y* direction). with initial shear modulus G = 1 kPa. The grid is gradually refined to 7 x 7 x 7 and eventually to 13 x 13 x 13 grid (2197 parameters, with resolution of *1.67 mm* in *x* and *z* directions and *2.5 mm* in *y* direction, which was sufficient to capture the variation of modulus for this particular problem). Measurements at 100 Hz are used first and then 200 Hz measurements are included (both with 30 dB Gaussian noise). The top panel shows the inverted G; bottom shows the true G. Planes 1, 2, 3 are the SWE measurement planes. Existing SWE methods image only in these planes, but our method images the entire volume, including outside these SWE planes. The root mean squared-error (RMS) error in the volumetric image is less than 0.7%, with the image on horizontal plane 4 is shown to illustrate the accuracy. This clearly illustrates the effectiveness of the proposed algorithm for 3D elasticity imaging.

We then conducted validation on a gelatin phantom with cylindrical inclusion. We perform elasticity imaging on a 2D version of FWI (Fig. 2). Particle velocity measurements on a dotted line are utilized (analogous to one of the three vertical planes in Fig. 1), to image the 2D region (analogous to the entire volume in Fig. 1). The two white dots represent two ARF pushes acting normal to the imaging plane. The true image is a cylindrical inclusion with dashed white boundary, with G = ~20 kPa, in a background with G = ~8 kPa.

The results demonstrate the high potential of the proposed FWI framework for applications such as 3D tumor elastography with the conventional 2D SWE scanners.

References

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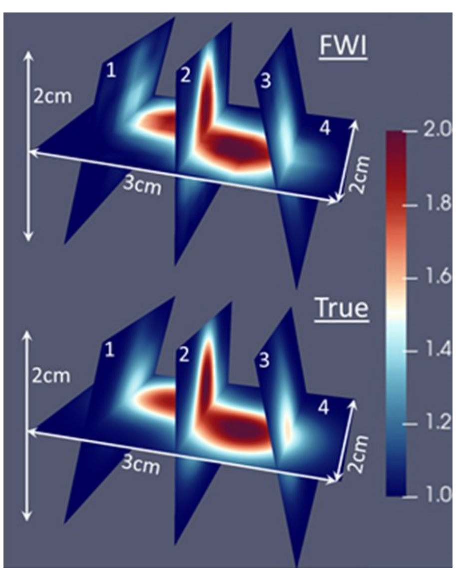

Fig 1. Example result from 3D imaging of diffuse disease. FWI is not only able to capture the variation in the measurement planes 1, 2, 3 but also the entire volume (elevational plane 4 is shown for illustration).

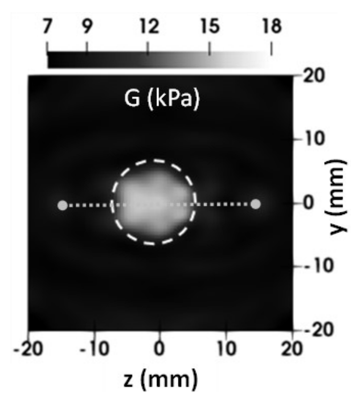

Fig 2. Validation of 2D version of the proposed FWI: the reconstructed image of a cylindrical phantom. Dashed line represents the reconstructed phantom by the dotted line represents the measurement plane.