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Materialized Action: Reformulating the "Doing of" Math Through Fiber Crafting

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ABSTRACT

This article examines how fiber crafting as a category of activity can develop mathematics learning and the conditions under which various fiber crafting traditions differentially cultivate mathematical understanding. Modifying the constructionist paradigm with relational materialist principles, this paper advances the notion of "materialized action," which describes the natural inquiry process that results through emergent patterns between learners and the materialized traces of their actions. This paper takes a qualitative approach, combining a design and intervention phase to look closely across a set of materials (i.e., three fiber crafts, knitting, crochet, and pleating) and engagement in a "powerful idea" (i.e., the role of unitizing in multiplicative proportional reasoning), as instantiated across three youth case studies, and as an illustration of how we can better understand micro-developmental learning processes. We identified three levels of unitizing that make up the larger idea of enacting proportional reasoning (PR) through materialized action, which build and catalyze toward one another and support emergent understanding of PR from the intra-action of the material and the learner. In their engagement with PR, youth employed different strategies based on personal choice, affordances of the materials, and practices of the crafting traditions. Materialized actions as a theoretical advancement has the potential to reformulate what counts as mathematics and can guide the design of mathematics learning that is embracing (rather than reducing) worldly concreteness in learning key domain ideas, with implications for the design of more equitable learning environments.

Introduction

Fiber crafting concerns represents a complex process of translating abstract patterns into embodied, systematic actions that dynamically integrate multiple sensory and physical elements: hands, needles, fiber (e.g., yarn), visual perception, muscular tension, and rhythmic movement. These intertwined actions involve the continuous construction and reconstruction of discrete units that, when strategically assembled, generate intricate patterns (e.g., Wertheim, 2005). Crafters sustain and explore their patterned action through deeply personally material choices—carefully choosing fabric, thread characteristics, thickness, color, and myriad other esthetic or functional variables. While constructionism (Harel & Papert, 1991) offers valuable insights into learning through artifact creation and "objects to think with," *the nuanced processes through which materials, actions, and development intertwine* remain under-developed and under-theorized.

Traditional constructionist perspectives have acknowledged bodily engagement through what Papert called *body syntonicity*, which posits that individuals imaginatively embody the objects they manipulate and design (Papert, 1980). Connecting physically with ideas that came from their

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mind, learners deepen their conceptual understanding as well as multiple epistemological ways in which concepts can express themselves in the world (Keune, 2022a).

In this study, we extend constructionist material perspectives by introducing the concept of materialized action—a novel analytical framework that foregrounds the generative power of materiality. Materialized action creates a developmental environment where learners generate multiple conceptual understandings by actively constructing knowledge through hands-on project work. This approach necessitates learners' exploration, deduction, and performative engagement with domain-specific ideas across interconnected modalities. We conceptualize materialized action as a micro-developmental condition that generates diverse epistemological approaches to understanding. Through materialized actions—defined by the crafted production of projects that demand constructive and deductive engagement with domain ideas—a rich spectrum of epistemic approaches emerges. In this paper, we illustrate this process through the specific domain of multiplicative proportional reasoning, demonstrating how material practices can transform abstract conceptual learning.

At its core, materialized action can be conceptualized as the patterns of intra-action (Barad, 2003) of learner(s) and material(s) in the construction of an artifact. While deeply rooted in the constructionist paradigm—which posits that creating and sharing externalized artifacts is particularly conducive to learning—materialized action critically extends this perspective by moving beyond individual cognitive processes. Instead, it focuses on the emergent, dynamic interactions that unfold between learners and the material traces of their actions. This theoretical approach bridges constructionism with new materialist perspectives, revealing how learning is not solely an individual cognitive process, but a collaborative negotiation between human agency and material affordances. Fiber crafting becomes a particularly rich site for investigating this dynamic, as it exemplifies how material properties actively shape and transform learning processes. The materialist lens challenges traditional educational approaches by positioning materials not as passive tools, but as active co-constructors of knowledge.

Mathematics learning, often perceived as an abstract, purely cognitive endeavor, becomes reimagined through materialized action as an embodied, material-discursive practice. Fiber crafts offer a unique lens for this investigation because they inherently involve mathematical concepts such as proportionality, spatial reasoning, and pattern generation. The very act of crafting—with its precise measurements, repeated units, and structural transformations—becomes a mode of mathematical thinking that is simultaneously concrete and conceptual. For instance, in the opening vignette, though both projects made use of the same type of yarn fiber (i.e., cotton) and followed the same exact pattern, it was the weight or thickness of the yarn and the differently sized crochet hooks that contributed to the difference in size. It is in fact the materials that shape the size and look of the final product and, as such, in tandem with the crafter, create the final product. Our study is motivated by a critical gap in understanding how learning—particularly mathematical learning—emerges through material engagement. By examining three distinct fiber crafting traditions, we seek to illuminate the *generative potential of material practices* in mathematical understanding, demonstrate how different crafting traditions *differentially cultivate mathematical reasoning,* and challenge traditional cognitive models of learning by foregrounding material agency. Specifically, we address two interrelated research questions:

1. How does fiber crafting develop opportunities for mathematics learning?
2. What are the conditions under which disparate fiber crafting traditions differentially cultivate opportunities for mathematical understanding?

This qualitative study captures a range of three fiber crafting activities that engage young learners in mathematical learning through personally meaningful design. This effort combines research on the use of textile crafts for learning advanced mathematics (e.g., Belcastro & Yackel, 2011; Greenfield & Childs, 1977; Harris, 1997; Peppler et al., 2022; Wertheim, 2005) with a relational materialist lens on learning and development (Hultman & Lenz Taguchi, 2010) to capture, analyze, and theorize how materials prompt human development and learning. As part of a longer-term qualitative study that focused on capturing evidence of learning via fiber crafts (Keune, 2024; Peppler et al., 2020; Saxena et al., 2023; Thompson, 2024), this paper presents close analysis of the entangled micro-developmental engagement of fiber crafts and three youth case studies to show how material changes led to engagement in powerful mathematical ideas of unitizing and proportional relationships during an introductory workshop experience. We chose three fiber crafts (i.e., knitting, crochet, and pleating) to illustrate how materialized actions across the crafts demonstrated similar patterns of emergence, yet engaged the youth differently in terms of mathematical engagement, and encouraged us to reconsider how we recognize mathematical understanding and enacting. Through this examination, we aim to reframe mathematics learning as an embodied, material process—one where knowledge is not transmitted, but dynamically co-constructed through the intricate dance of human intention and material responsiveness, with the potential to disrupt inequitable mathematics education assumptions and practices.

Discerning what remains the same and what changes is a key mathematical practice. The Japanese art and craft ofamigurumi, the creation of small hand-knitted or crocheted toys, reflects this tenet by illuminating the importance of materialsand their impact on the overall look and dimension of the final product. In her amigurumi explorations toward the creation oftoys for her cat, one of the authors felt firsthand this material importance. Figure 1 shows an example of two amigurumiprojects, which are different in size, but appear to represent the same object, a hand-crafted whale. At first glance, it mightappear that what is different is the pattern that was used to create the toys; one is evidently larger in size and appears to havea greater number of stitches in its respectively lengthier circumference. However, both toys followed the same instructions and-pattern: A magic circle (MC) with 6 single-crochet (SC) stitches, followed by an increasing and then decreasing number of SCstitches in subsequent rows to maintain the spherical shape of the toy (12 in the second row and 18 in the third), exemplifying proportional growth. Why is it, then, that one ended up significantly larger than the other?

Figure 1. Hand-crafted whales that are different in size.

Background

Math and craft

Historically, math learning and fiber craft learning were placed in opposition to each other in European and American schools, with boys learning higher math while girls learned textile crafts, such as sewing and embroidery (Harris, 1997). However, researchers have observed ample connections between textile crafts and math that belie this socio-cultural and historical separation, such as in knitting, crochet, cross-stitch, quilting, needlepoint, and tatting, among others (e.g., Belcastro & Yackel, 2007; 2011; Harris, 1997). The first technique to successfully model hyperbolic planes in physical space was crochet (Taimina, 2009; Wertheim, 2005) and the first modern computer was based on Jacquard's automatic punch-card loom (e.g., Essinger, 2004). In fact, it has been argued that weaving leads to highly mathematical engagement in cultural and educational contexts (Greenfield & Childs, 1977; Peppler et al., 2020; Saxe & Gearhart, 1990; Thompson, 2019, 2020). For example, through ethnographic research, Greenfield and her team presented that weaving can be a tool for developing cognitive skills related to pattern recognition, spatial transformation, and meta-representational skills (Greenfield et al., 2003; Maynard & Greenfield, 2003).

Other work has demonstrated mathematical learning through textile craft engagement, such as sewing of tents and costumes, knitting, crochet, and weaving in both in-school (Peppler et al., 2018; 2019) and out-of-school contexts (Bender & Peppler, 2019; Peppler et al., 2020). Beyond mathematics, fiber crafts have been an inspiration for technological inventions (e.g., Hofmann et al., 2019; Igarashi et al., 2008; Keune et al., in progress) and computer science learning (e.g., Keune, 2023, 2022a). Particularly the repeated and rhythmic movements of people and craft materials that are connected to mathematical "doing" present opportunities for reimagining education (Keune, 2022b).

Inclusive materialism, proportional reasoning, and unitizing in mathematics

In the constructionist tradition, researchers look for powerful ideas that are part of the domain that are persistently difficult as taught using traditional approaches. One such powerful idea, which we examine in this study, is the role of unitizing in multiplicative proportional reasoning (PR). Proportional reasoning is the understanding of the multiplicative part-whole relations between rational quantities (de la Torre et al., 2013) and is a predictor of future mathematics achievement (Behr et al., 1992; Boyer & Levine, 2015). While PR has applications in a range of professions, with practitioners using it in their daily practice (Noss et al., 2000), it has persistently been challenging to learn (Lobato & Thanheiser, 2002); often, young learners try to use additive instead of multiplicative strategies (e.g., incorrectly solving for x in $\frac{2}{3} = \frac{x}{6}$ by adding 3 to both numerator and denominator instead of multiplying both numerator and denominator by 2; Hart, 1981; Lin, 1991; Tourniaire & Pulos, 1985; Van Dooren et al., 2010). Nonetheless, children as young as 5 or 6 can reason proportionally, as well as develop intuitive proportional reasoning strategies, if spatial-perceptual representational problem formats are used (Boyer & Levine, 2015).

Unitizing is a foundational concept for multiplicative and proportional reasoning. Götze and Baiker (2021) found that a language-responsive introduction to multiplication as unitizing (e.g., introducing 3×4 as "having 3 fours") helped students to improve their multiplicative reasoning. Additionally, a study of children's PR before formal instruction in ratio and proportion found that unitizing was essential for developing PR: "[I]t is useful to view a ratio as a unit, the result of multiple compositions of composite units" (Lamon, 1993, p. 58). Lamon (1996) continued her work on unitizing with a study that explored the development of unitizing in youths' partitioning strategies; she found that more sophisticated unitizing (partitioning into composite units) occurs over time and is difficult for children to develop.

Yet, there is an alternative way of looking at proportional reasoning. Within their work on Inclusive Materialism, de Freitas and Sinclair (2020) argue there is a powerful possibility in feeling and practicing units across multiple materialities as to "disturb narrow (and perhaps white, western, male) images of mathematics—and to open up opportunities for a more pluralist school mathematics," that draws on different cultural experiences, materialities, and abilities (2020, p.2). For instance, in their crafting practices, the Yup'ik of Alaska make use of proportion in their initial measurements by standing in a line and estimating measurements relative to one another's height (de Freitas & Sinclair, 2020). Additionally, de Freitas and Sinclair (2020) bring into focus tangible materials that can streamline better understanding of these concepts (e.g., rope stretcher with knots at equal intervals). Adding to this work, we consider how units can be dynamic and tangibly produced (rather than being given pre-formed units by the teacher/problem) as well as how units sediment and build over time (e.g., stitch unit to patterning unit to project unit).

In sum, we aim to illuminate the processes of epistemic engagement that learners embark on during constructionist experiences, using the powerful idea of unitizing within PR to explore how disparate materials and their associated crafting practices differentially cultivate mathematical understanding. This work takes a view of unitizing as a production-centered process shaped by youth's individual approach to the craft and the specific materials in use that affect the final product. In this paper, we seek to uncover whether a range of "materialized actions" resulting from co-constructions across various crafting traditions (e.g., knitting, crochet, and pleating) in relation to the same domain (e.g., proportional reasoning and unitizing) can present an understanding of the domain concept as something that is anchored in the physical world, as well as surface new understandings about the domain and how it can be expressed.

Constructionism, objects-to-think-with, and body syntonicity

This study takes as a starting point the theory of constructionism, which posits that learning occurs best when individuals design physical (or digital) constructions that can be shared, and by that represent cognitive transformations that happen as learners actively engage with domain-relevant ideas. Working out reasons for why designs fail and adjusting designs to address such issues is one important way to deepen understanding of the mental models and concepts employed in design (Kafai, 2006; Kolodner et al., 2003; Litts et al., 2016; Papert, 1980). This iterative approach turns materials and tools–physical or digital–into *objects-to-think-with* that provide opportunities to improve upon and change mental models through design modifications and reflection (Bamberger, 2014; Papert, 1980).

Attention paid to the types of materials used for learning is not without consequence, as materials, and the relative marginalization of other materials, have shown to shape domains in formative ways. For instance, Michael Friedman (2018) detailed how the compass and straight-edge used since Greek antiquity produced a range of mathematical techniques and practices that subverted and marginalized other mathematical principles based on the folding of parchment. This has manifold consequences for how we conceive of mathematics today. Similarly, the epistemological consequences of sidestepping Fröbel's folding, embroidery and sewing "gifts" as materials for exploratory patterning (Brosterman, 1997; Fröbel, 1885; Kafai et al., 2010) looms large in our conception of the mathematics domain and how it is experienced by learners. As there is not "one way" to experience the domain of mathematics, such observations lead us to consider: a) The underexplored areas of the domain that rest on actions and practices afforded by non-dominant materials and their patterns of activity; b) the nuanced understandings of the domain that they can provide; and c) what is ignored within the domain by their absence. In short, the material aspect of objects-to-think-with is important for understanding learning processes, specifically in the ways in which co-creations between materials and learners lead to new understandings that are not seen as representative of how a domain is traditionally expressed.

In addition to material considerations, constructionism has also taken the body and its movements into account through what Papert called *body syntonicity,* which considers the body (and its movements) as material by frequently imagining or mapping the body onto the object that is manipulated (Papert, 1980). As an illustrative and often cited example, Papert's robotic LOGO turtle physicalized code in 3D environments. Children using the turtle could imagine the path it would take when instructed with certain code, and sometimes even enacted the path taken with their own bodies, connecting their programming notions to a host of epistemic understandings informed by physical movement. This is an important idea for developing ways to evidence materialized actions, which in this paper means the body movements involved in the production of units and their multiplications. The present paper also seeks to account for the materialized doing that extends this idea by following the second author's approach of merging constructionist ideas with material-focused theories and sharing this perspective next.

Materialized action for epistemic engagement

Correspondingly to body syntonicity, how learners interpret the traces that actions bear upon the materials they use (e.g., the tension of gears, the snap of magnets, the folds of origami) invites an investigation into *material* syntonicity: Recognizing how domain concepts are projected by the behaviors of material and their response to learner interaction (Keune, 2022b). Because of this, it is important to move beyond not just facilitating engaging learning experiences but understanding the opportunities for developing stronger epistemic understandings over time. These theoretical ideas (e.g., material syntonicity) stem from taking on dual theoretical perspectives that merge constructionist approaches to learning with posthumanist perspectives to understand how domain learning emerges and how the materials used are actively taking part in driving domain-specific learning (Keune, 2022b). This is important for the theoretical background of the present paper, because it begins to show that taking a dual theoretical perspective, which is combining constructionist ideas with material-focused theories, can advance our understanding of learning. This is especially important when we want to introduce nontraditional materials into mathematics learning as a means for broadening participation (as we are doing in the present paper) because the dual theoretical lens provides an opportunity to evidence the process in which mathematical ideas are engaged so that they can be named and recognized beyond individual instances.

In the present study, we advance this trajectory by drawing on relational materialist views (c.f., Hultman & Lenz Taguchi, 2010; Lenz Taguchi, 2011) that call for lowering and, in the best case, flattening hierarchies between people and things and, thus, making it possible to interrogate the relationship between materials and learners by questioning typical assumptions in educational research that materials serve people through the mediation of concepts. This leads to a focus on intra-actions (Barad, 2003), the actions that emerge as people and things form relationships with one another. This contrasts with a focus on separate individual parts and shifts attention to the production of something more than the individual parts.

From this approach, learning can be considered the formation of relationships through which potentially unforeseen possibilities are being produced (Hultman & Lenz Taguchi, 2010; Lenz Taguchi, 2011; Keune & Peppler, 2019). Prior work that examined maker-centered learning in the context of additive manufacturing in out-of-school settings as well as fiber crafts as a context for computational learning showed that this view naturally expands constructionist notions toward acknowledging how people and things dynamically emerge together, and how learning trajectories, learning activities, and learning environments expand beyond the planned (Keune & Peppler, 2019). The prior work in the domains of computing and additive manufacturing showed in larger scale and microanalytic ways that studying the forming of relationships among materials and people can shift not only theoretical understanding of learning in context but can also impact educational practice toward more inclusive and equitable approaches within domains that are still marked by inequitable participation (e.g., Keune, 2022b; Keune & Peppler, 2019). The work presents that the study of domain learning should focus beyond learners' and educators' intentions–there is a field of research around the equal relationships between learner and material that can shift domain learning approaches.

Following this dual theoretical take, in this study we conceptualize the notion of materialized actions as a process of producing and constructing small elements that together make up the performance of what we can materially recognize as an epistemic idea. We show this by first aligning three crafts with the mathematical idea of multiplicative proportional reasoning and evidencing how this epistemic idea is engaged and constructed in the crafts. Then, we show how young people produce the idea in their own projects to evidence a way to understand and value mathematical epistemic engagement in relation to relevant yet nontraditional tangible material contexts. Materialized action presents how knowledge engagement through crafting necessitates the performance of mathematical ideas that we aligned the crafts with (i.e., multiple proportional reasoning).

Methods

We designed crafting activities that built on a grounded understanding of the potential of crafts to cultivate mathematical learning, informed by prior embedded ethnographies as well as interviews with educators and professional crafters (Peppler et al., 2022). Specifically, we considered which features of our designs (i.e., aspects of the tangible manipulatives and/or aspects of the supporting activity) appear to support mathematical learning. To accomplish this, we embarked on a qualitative study with the intention to inform activity design across three fiber crafts with considerations of what features of activities could lead to exploration of PR, and then evaluated these features as part of two iterations: 1) artifact analysis with a team of adult crafters to further align activities with PR learning formally, and 2) a three-day fiber crafts camp in which youth performed the three craft activities to understand how the crafts supported engagement with PR.

To support the engagement with PR through each crafting tradition, we co-designed activities with five adult crafters to craft artifacts, design activities, and analyze mathematical concepts within craft products. One crafter had been knitting for two years and had helped with different aspects of this project previously. The second crafter was an undergraduate researcher on the project who had been practicing all sorts of textile crafts for at least five years and had made many example projects for us. The third crafter was also an undergraduate researcher who was majoring in fashion design and helped by sewing example projects. The remaining two crafters also had experience in mathematics; one had recently graduated with an MS in mathematics, and the other was in a mathematics education graduate program. Both mathematicians were also excited about craft, had experience with crafts, and made their own crafts to sell at local craft fairs.

With regards to the crafts camp, we captured detailed accounts of the crafts through photographs of projects. We also captured youths' verbal expressions and detailed accounts of their physical engagement with the crafts and their materials through video recordings. We used qualitative methods of artifact analysis (Pahl & Rowsell, 2010) of crafts to analyze activities for mathematics alignment. We then performed modal analysis (Abrahamson, 2009) of how the youths' bodies performed the craft activities and the PR embedded within to better understand educational productions at play at the craft tables and to identify the materialized aspects of the craft related to units and shapes that added to PR engagement. We argue that these approaches are productive for better understanding mathematics learning and engagement with tangible tools and materials, as they support the understanding of aligning crafts with mathematics concepts as well as understanding emergent meaning-making with materials designed for mathematical engagement.

Settings

The setting of this research was a *public library of a midwestern college town*, where we first piloted and later facilitated a three session-long math-based fiber-crafts summer camp in the summer of 2017. To pilot the three activities of the craft camp—knitting, crochet, and sewing—we facilitated two drop-in sessions per craft at the library's crafting space for 12–19-year-olds. The pilots supported the refinement of the facilitation strategies as well as participant recruitment for the summer camp, as the camp was facilitated in the same library. We offered the camp in the library's activity room located on the ground floor. The room included tables and chairs that we arranged into small group table pods. The camp took place over the course of three consecutive days. Each day's session was four hours long, and on each day, youth learned a new craft and created a project with the craft: 1) Crochet a circular bag, 2) knit a bag out of a rectangle, and 3) sew a pleated pattern into a bag. Time was given to learn the basics of the craft and then to make the project.

Participants

We targeted middle-school-aged youth for the crafting camp because this is the age at which PR is typically taught (Common Core State Standards Initiative, n.d.) as well as the age at which girls—and other underrepresented youth—begin to wind down on interest with STEM (Corbett & Hill, 2015). The library supported the recruitment efforts by designing and printing flyers that we distributed in local shops. A total of 15 participants registered for the workshop and two additional participants dropped in. Due to attendance variations, a total of 17 youth attended, but 15 per day. All registered participants were between 9–12 years old and of all participants, 16 were female and one was male. Two participants were joined by their parents to support language translation.

During the camp, we asked youth to form small groups and to distribute themselves across the arranged craft tables. Each craft table had a dedicated adult facilitator. Throughout the camp, the youth had to learn a new craft at the start of each session. During the crochet and knitting days, three facilitators worked with craft tables of 4–7 youth, and on the pleating day, four facilitators worked with craft tables of 3–4 youth. Additionally, during all three days, two adult facilitators were assigned to float around the room and to provide just-in-time help with crafting, checking of camera functions, as well as taking observational notes on the flow of the day's activities.

For analytical purposes, we focused on *three focal youth and their experiences with PR through the crafts*. Across the data, we chose exemplary cases to understand the conditions under which PR arises. While we looked at the youths' experiences across the crafts, we predominantly focused on one craft experience per youth. Youth had to actively participate in the crafts and create projects that met the instructional requirements so we could investigate their experiences with proportional reasoning. Another criterion for the focal youth selection was that youth had to work on projects with low to medium facilitation to ensure that most of their projects were created by them rather than the instructor. Youth with low facilitation needs could complete their project with initial instructions from an adult facilitator, while those with medium facilitation needs asked facilitators for instructions throughout the project progression but did not hand their project over to facilitators. Youth with high facilitation needs asked facilitators to start or complete youth projects and cases in which adult facilitators implemented most of the projects. Of all, 13 youth worked independently with low facilitation needs.

Given the early-stage explorations of the activities within educational settings, we aimed to analyze exemplary youth engagement because we were interested in investigating the conditions under which PR engagement was supported and came about. Of the 13 with low facilitation needs, eight youth followed the design activity instructions per craft.

For knitting, we selected a youth named Katie (all names are pseudonyms), who created a knitted bag with differently colored yarn and explored PR by restarting her project and comparing it to other knitting techniques. For crochet, we selected a youth named Tracy, who created a three-row circular bag with pink chunky yarn and engaged with PR through unitizing and stitch distribution. For pleating, we selected a youth named Margaret, who created a bag with two pleats using a fabric with a planetary print and engaged with PR through repeated engagement and by inventing strategies to make her proportional units permanent.

Data sources

We first created *example projects* with all three crafts that acted as proofs of concept for the initial activities. The example projects served as reference materials that guided our explorations of the underlying math concepts and shaped our understanding of how PR was instantiated in each of the crafts. The fiber projects were used in this inquiry process to pull apart, look closely, and reconstruct soft constructions.

Additionally, the data sources included *video recordings of the youth camp* to observe the youth-produced proportional reasoning across projects through material unitizing and shaping. The cameras captured the youth working on their projects and any audio as the youth and adults spoke. We captured 40 hours of video data (i.e., four hours per camera per day). Following Derry et al. (2010), we set up video cameras facing the youth and captured their hands and faces as they worked on their projects, filming each table group. Thus, based on the number of table groups per craft, we captured the crochet and knitting days with three cameras and the pleating day with four cameras. Throughout the camp, at intermittent points in their design process, facilitators asked youth about what they were doing and what else they could try.

Lastly, the data sources included *231 photographs of youth projects*. We used phone or iPad cameras to capture pictures that showed the dimensionality of the artifacts and details of the youth's projects to support reverse engineering of the steps the youth performed. The photographs provided a more detailed view of the projects when videos did not support clear images. We linked the photographs of the projects to the youth participants who created them to keep a detailed record of the youths' mathematics and craft engagement while also allowing us to return the project to youth at the end of every day.

Analytical techniques

We first *analyzed the example projects* generated by the research team with the adult crafters. Our analysis included three layers that carved out connections between crafts and PR: 1) Verbal descriptions, 2) visual representations, and 3) corresponding mathematical notation. The abstraction supported the refinement of the activity design in that it helped narrow down which affordances and features of the designed activity were necessary to support the engagement with PR. For the verbal descriptions of PR, we generated step-by-step breakdowns of the process from the start to finish of the craft project. Next, we identified rules that could be abstracted, including steps that occurred several times in the same way or with slight variations. We created visual representations (e.g., illustrations or photographs) that showed the emergent material and craft pattern and further abstracted the activities toward symbolic mathematical representations. This layer of analysis focused on intersections of math and particular aspects of the crafts (e.g., distribution of crochet and knit stitch units) and how these aspects differed across crafts. Lastly, we represented the rules that governed the patterns we designed through mathematical notation which directly connected PR with the designed craft activities. In doing so, we took apart and reconstructed the material projects continuously to guide the translation to mathematical notation. As per constructionist philosophy, the abstractions into mathematical notation were not a part of the camp experience—at no point during the workshop was the learning of PR an explicit goal of participation. Rather, to do each activity well, the learner had to successfully execute PR concepts, even if they were never described to the learner as such. Instead, we abstract the learners' projects into mathematical forms after the fact to illustrate the intersections of canonical mathematical doing with crafting, which is not canonical of mathematical doing (see also Keune, 2022b). Such translations rarely take place explicitly, and especially in our camp, the youth *felt* the materialized units and their proportional relationship as the units were multiplied, rather than calculating, for instance, amounts of stitches per row.

The analysis of the video recordings of the youth camp focused on how youth produce PR across fiber crafts through material unitizing and shaping. We were particularly interested in how crafters produced units in the materials—and the subsequent changes to the shape of their project—as well as how the unitizing and shaping added to the material unit and shape production. To identify relevant moments of the focal youths' craft engagement, we viewed all four hours of video data for all focal youth. This included creating content logs of the video that summarized the focal youths' practice in relation to the identified mathematics for every 5–10-minute-long blocks of video. We used the summaries to highlight where in the video the most relevant moments of proportional reasoning happened. These were the moments that we focused on for further in-depth analysis. Our analytical process was inspired by relational materialist understanding that took into consideration the role of the body in mathematical learning experiences. Accordingly, our analysis shifted away from interpreting the children's intentional design meanings, instead focusing on *how the materials and bodily actions collaboratively produced proportional relationships through the crafting process.*

Our analysis also drew on the photographs of youth projects, which showed their material productions of proportional relationships across crafts. The images provided the possibility to closely observe stitches and folds in order to reconstruct the mathematical doing that occurred to produce the project. The photographs and detailed videos of completed projects were used to trace the production processes. We particularly focused on the differences between planned and implemented projects (e.g., in relation to stitch size) as reference for mathematical processes.

Findings

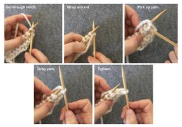
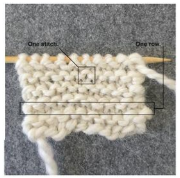
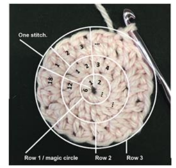
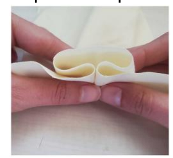
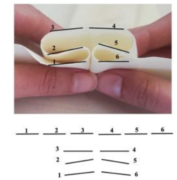
To uncover how fiber crafting develops mathematics learning, as well as the conditions under which disparate fiber crafting traditions differentially cultivate mathematical understanding, we present the three levels of unitizing that make up the larger idea of enacting PR through materialized action. In this analysis, we present how unitizing levels build and catalyze toward the other through crafting, and how understanding emerges from the intra-action of the material and the learner. Our analysis centers on how participants reason with the stratification of personally defined units throughout the development of their crafts.

Unitizing and proportional reasoning per crafting tradition

In contrast to the use of established units as the basis for ratios and proportional relationships, fiber crafting begins with an initial stitch unit that users define through their choice of materials and their body's relationship to their manipulation. As they knit, crochet, or pleat, crafters reason with multiplicative part-whole relations as rhythmic and repeated movements of people and materials arrange and multiply stitch units into pattern units, which are multiplied again into a project unit. In this study, we define three levels of unitizing:

> Stitch unit (i.e., knitted stitch unit, crocheted stitch unit, fold unit)—The unit basis for a project, consisting of several small intra-actions between people and materials specific to each crafting tradition (see Table 1). Stitch units form the basis of proportional relationships when considering the number of stitches per row (i.e., knitting), stitches per sector in a circle (i.e., crochet), or inches per pleat (i.e., pleating).

Table 1. Overview of unitizing and proportional reasoning per craft activity (i.e., project units).

Project units					
Knitted bag: A flat rectangle shape folded in half.		Crocheted bag: A flat circle with walls.		Pleated bag: A rectangle folded into a pleated square.	
					
Knitted stitch unit: Go through stitch, wrap around, pick up yarn, drop yarn, tighten. Knitters build a unit that is multiplied to produce pattern units.	PR is called for through gauge calculation (i.e., stitches per horizontal inch and rows per vertical inch). To understand gauge, knitters add rows of multiplied units.	Crocheted stitch unit: Go through the stitch, yarn over, go through stitch, yarn over, go through one, yarn over, go through two. To understand proportional growth, crocheters increase the number of stitches per row, which multiply to form pattern units.	PR is called for when looking across sectors per row and the within-sector relationship of proportional growth. More specifically, each stitch in row 1 corresponds to 2 stitches in row 2 and to 3 stitches in row 3.	Fold unit: mark width of the pleat, draft score marks at half width of pleat on both sides of the pleat, fold fabric at four markers.	PR is called for in each individual pleat (i.e., inches per pleat) which come into relation to produce pattern units. Pleaters pinch up and flatten layers of fabric to create a pleat. Pleaters have to mentally visualize layers of fabric that are hidden from sight.
Stitches: $\frac{x \ \ stitches}{21 \ \ horizontal \ \ cm} = \frac{15}{10\,cm}$ Rows: $\frac{x \ \ rows}{6 \ \ vertical \ \ m} = \frac{8}{4\,cm}$		Row: Number of Stitches 1:6 = 2:12 = 3:18		Total length of fabric required: L = l + 2np where *l* stands for the length of the fabric in the outcome, *n* for the number of pleats, and *p* for the pleat width. Unfolded: Folded pleat = 3:1 = 6:2	

Pattern unit—The repetition of stitches or folds into larger patterns to define the project's overall size (e.g., the number of stitches per horizontal inch, crochet stitches per row, or inches per pleat). By bringing stitch units in relation, patterns emerge. Note: We define a pattern as a form or model used for imitation, not to be confused with a crafting "pattern," which is crafting parlance for a physical diagram users follow (or pieces users trace, in sewing) to make a project.

Project unit—The combination of completed patterns that form the finished product. Where the mathematical characteristics of a materialized action begin to emerge from the two units above, the project unit shows the mathematical connections even more clearly.

In the next section, we walk through how each craft uniquely supports unitizing within PR, and investigate the processes of epistemic engagement that learners embark on within each crafting practice. Guided by de Freitas and Sinclair (2020) decolonial approach toward units to uncover dis/abilities in curricular frameworks, we zoom in on the materiality of units within proportional reasoning as practicing routines and how they build up a recognizable unit that can be chained to produce a measurement, which, in turn can then be put into a proportional relationship.

How fiber crafts differentially develop mathematics learning

In the crafting camp, youth formed relationships with domain concepts (i.e., unitizing and proportional reasoning) through their iterative engagement with the materials as they crafted in preferred ways (e.g., by stitching and unraveling, by exploring the number of stitches per row). In our analysis, we examine how the material construction as part of the craft brought about engagement with PR, which took a different form for the three youth and employed different strategies based on personal choice, affordances of the materials, and practices of the crafting traditions.

Katie's knitted bag: Stitching a unit and PR as stitches and rows per inch

For the knitting activity, participants learned how to knit a bag composed of two individually knitted squares to be later sewn together, each approximately 4×4 inches in size. First, participants learned how to cast on stitches as well as execute *knit* and *purl* stitches (i.e., the fundamental knit stitches). Second, with their choice of yarn and knitting needles, participants practiced casting on a set number of stitches and knitting for a few rows to get a sense of the proportional relationship between stitch length and stitch height in order to define their gauge (i.e., a small knit that gives a sense of the proportional relationship produced by knitter, needle size, yarn thickness, stitch unit, pattern unit, and the resulting tension from the interplay of the human and non-human parts). Third, participants completed their projects by being taught how to cast off stitches, which provided a smooth edge to their knit.

When knitting, stitches become routines that form a unit from which the proportional relationship can emerge. One stitch unit consists of the following actions that involve the coordination of hands, needles, yarn, eyes, tension, and speed: Go through stitch, wrap-around, pick up yarn, drop yarn, and tighten (see Table 1). The action that the hand, yarn, needle, eyes, etc., perform together to produce a unit has an imprint on the material (e.g., how the materials respond to manipulation) and the crafter (e.g., how their hands develop muscle memory and the consistent tension of yarn when wrapped around fingers). Stitches' dimensions can vary in proportion because the length and height of one stitch depends on the knitter's personal tension or grip of the yarn, the selected needle, and the yarn itself. It is an interplay of people and materials that produces a stitch unit, although the steps of the routine stay largely the same.

One of the youth was Katie, a 10-year-old experienced knitter who showed others how to knit. Katie produced stitch units only to unravel them and to restart a total of 12 times, working to establish a consistent and intuitive feel for her stitch units, including determining which yarn thickness and needle size to use. The size and tension of her stitch unit varied across iterations. Where initial stitch units were loose and irregularly shaped, as Katie got into a routine, her stitch units became tighter and more uniform. Katie and a neighboring youth also explored stitch units through a conversation about arm knitting, a knitting technique that uses the arms of the crafter in place of knitting needles (see Table 2). Together, the youth determined that the stitches would be gathered on one arm and picked up by the other arm (see Table 2, panel 1 and 2). Through gestures, the youth compared the effects of using different materials (i.e., wooden needles vs. arms as needles) on one's personal stitch unit (see Table 2, panel 3 to 5). Through her body posture and arm gestures, Katie expressed how the size of a needle affected the amount of yarn needed for a stitch as well as the size of a stitch unit (see Table 2, panel 6).

Starting over also made it possible for Katie to practice and develop a sense of her personal gauge, reflective of pattern units. With an increasing number of unraveled projects, Katie began to consider how the number of stitches she cast on would relate to the end product's size, counting the stitches she cast on her needle. Additionally, Katie also compared knitting techniques to

Table 2. Transcript of a conversation about arm knitting that involves stitch units and pattern units.

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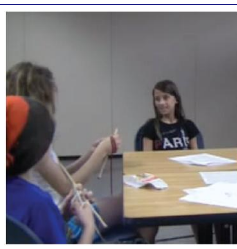
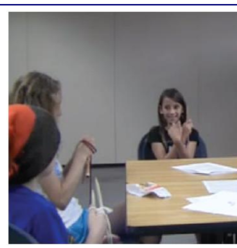
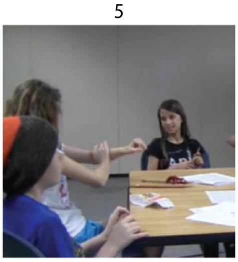
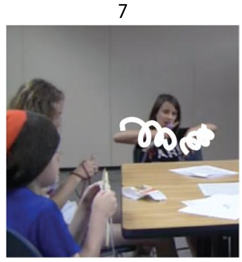
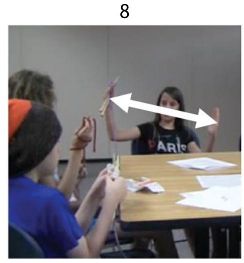
			
Katie: "Your arms are like needles."	*Sarah: "Like this?"*	*Katie: "I don't know how exactly."*	*Katie: "Now that I think about it, it's like the needles."*
Katie turns to Sarah and lifts her arms.	Sarah lifts her project. Katie drops her arms and nods.	Katie lifts her arms and twists them.	Katie picks up her project and points at the needles.
Katie's arms become needles.	Sarah introduces her project as a comparison.	With arms as needles, Katie explores how arm knitting would work.	Katie suggests that arm-needles would act similarly to wooden needles.
5	6	7	8
			
Sarah: "Ah." *Katie: "Yes."*	*Katie: "If you used this arm, it'd be stitches that big."*	*Katie: "Lalala"*	*Katie: "Then you have that much."*
Sarah lifts her left arm and grabs it with her right hand at three places. Katie nods. Both knit on.	Katie holds her hands one foot apart.	Singing, Katie waves her arms.	Katie holds her arm two feet apart.
Sarah's arm becomes a needle and the grabbing motion become stitches gathering on the needle.	Katie shows how the size of the project becomes larger with arms as needles.	The waves become stitches and Katie adds a few stitches to her imaginary project.	The imaginary project grew over twice in size and, thus, at a faster rate compared to using wooden needles.

get a physical sense of the size of a stitch in relation to the created pattern unit in space, and, more specifically, the length of a row of stitches (see Table 2, panel 7 and 8). For example, Katie discovered that six stitches in regular knitting is shorter than six stitches done in arm knitting. This is relevant because needle size is one aspect of how knitters conceive of their personal pattern unit, which ultimately shapes the look and size of a stitch unit (i.e., how big or how loose it is). As she worked, each stitch reconstructed the rectangular stitch unit that became the basis for a proportional relationship while this reconstruction was a part of forming the pattern unit. Each pattern of stitches thus formed another unit of the mathematical materialized action that left an imprint on Katie as she repeated units and combinations of units and the yarn that formed into the visible pattern shape.

Where the mathematical characteristics of a materialized action began to emerge from the two-unit levels above, the project unit showed the mathematical connections even more clearly. A knitted stitch unit is rectangular in shape and, thus, the stitch height is unequal to (shorter than) stitch length. This produces a proportional relationship, which in knitting looks like a performance centered on the gauge of a knit. As described above, to create a particular sized project, Katie first defined her stitch unit and then considered how many stitch units per row and how many rows she needed in total (see Figure 2). As she worked, Katie noticed the proportional relationship at the level of the project unit, when she realized her project unit did not match the drawn pattern unit and that her stitch unit was not square as she had anticipated in her sketches. For instance, Katie's pattern unit (i.e., project plan, see Figure 2) showed two squares of ten stitches per ten rows side by side, which assumed that a stitch unit is as tall as it is wide. Following her plan, Katie cast on 20 stitches.

By comparison, her implemented project unit included 8 rows: Two purple rows (one cast on, i.e., the stitches that Katie added to her needle at the start of the project and that defined the number of stitches each subsequent row would include), two pink rows, one blue row, and three green rows (one cast off). Even these eight knitted rows (instead of the planned 10 rows) produced a rectangle rather than a planned square. She adjusted her pattern unit so the drawn plan would represent the project unit with the colors she implemented (Figure 2, bottom left). Yet,

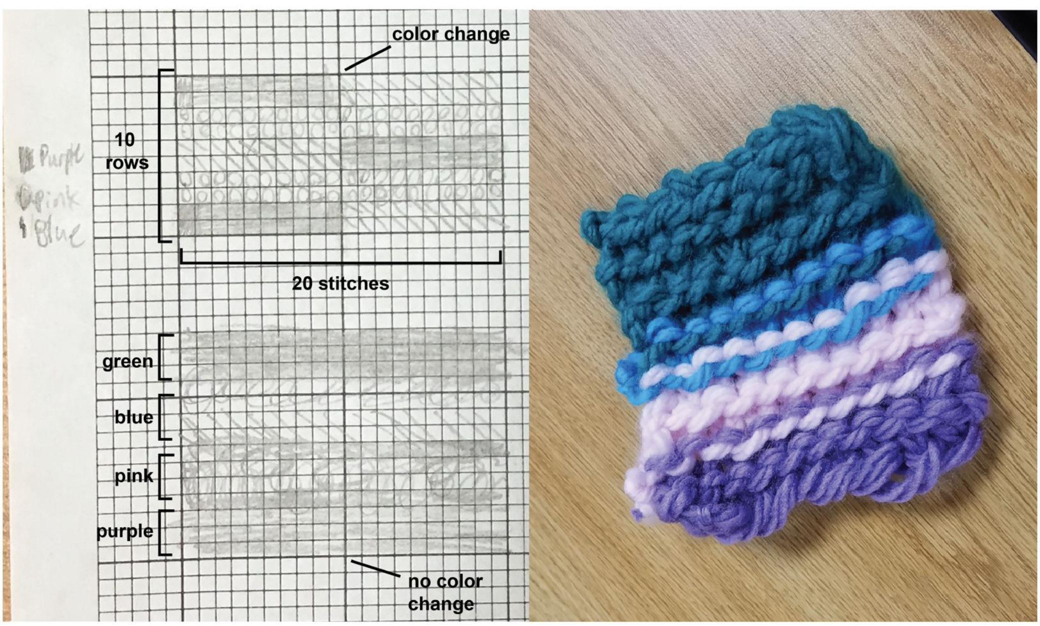


Figure 2. Sketched annotation of Katie's knitted project (top left), adjusted sketched annotation of Katie's knitted project (bottom left), and Katie's project (right).

when comparing the adjusted plan to her completed project, the plan included more rows per color than Katie actually knit and rows were drawn with less accuracy compared to the previous drawn pattern unit (see Figure 2, top left). This evidence shows that Katie discontinued considering the squares of the graph paper as individual stitch units and rather seemed to use the squares to illustrate the space the pattern unit with its colors would take up. Further, it indicates that Katie noticed that the number of rows she knit produced a project unit that was rectangular rather than square in shape and, therefore, that the individual stitch units were taller than they were wide. Thus, by moving across her sketched annotation (i.e., pattern unit) and her completed project (i.e., project unit), Katie discovered that the ratio of stitch width to stitch height is not 1:1. Instead of redoing her design, Katie adjusted her project unit based on the physical outcome that was different from the pattern unit; she said: "So this is going to be a little bit bigger than the square because this has a flap on it, too." Katie decided to use the extra space, resulting from the produced proportional relationship as a feature in her bag project unit (i.e., the length of her stitch unit to the project's length and the height of her stitch unit to the number of rows produced).

Repeated undoing and redoing of knitted stitch units and comparing how knitting techniques change the stitch unit's size helped Katie get a sense of her own gauge (i.e., the pattern unit) and the aspects that drove it. Knitting presented an opportunity for Katie to explore how combinations of stitch units (i.e., pattern units) could be arranged into a larger whole (i.e., project unit). This combination and the comparison of a drawn pattern unit with an implemented knitted project unit led to the exploration of proportional relationships—that is, numbers of stitch units per row. Moving across three different units provided Katie with the space for iterative material exploration (i.e., through the undoing and redoing of stitch units) and to make sense of the relationship across units, which brought about the implementation of proportional reasoning, but in greater complexity than what we would find in traditional classroom exercises.

Furthermore, we can see that learning about the epistemic idea is moving between units and is building toward larger constructions. This performative comparison of knitting with needles and knitting with arms was indicative of Katie's developing sense of a key material aspect of the craft that affected the production of a stitch unit, the basic element for PR within knitting. Yet, when moving from stitch unit to pattern unit, we start to see intersections and moving back and forth across units. Katie's stitch unit produced a new material that patterned units built on.

Tracy's crocheted bag: Stitching a unit and PR as stitches per sector and row

For the crochet activity, participants learned how to crochet a circular bag, which enacts ratio and proportion in a similar way to knitting by considering the number of stitches per row. However, the circular nature of the project in crochet also prompts a consideration of the number of stitches across sectors and the within-sector relationship of proportional growth. To create a flat circular shape, crocheters first create a *magic circle*, which includes creating a slipknot and crocheting *chain stitch* units (e.g., a series of looped stitches forming a chain-like pattern) into the knot's loop, with a desired number of stitches. The number of stitches that make up the magic circle defines the factor of multiplicative proportional growth, with each stitch representing one separate sector of the whole circle. By bringing stitches in relation with one another, patterns emerge that impact the material shape and dimensionality of the produced units. To maintain a flat circular shape, crafters produce and reproduce stitch units that are then uniformly distributed across sectors and rows. One of the youth, Tracy, a 9-year-old who was not very experienced with crochet at the start of camp, produced a magic circle composed initially of six crochet stitch units. Where in a rectangular crocheted project, stitch units resemble rectangles like in knitting, in a crocheted circle, inserting a stitch into the prior row's stitches pulls the yarn to produce a stitch unit resembling a trapezoid. Similarly to knitting, the overall shape and size of individual stitch units is co-constructed as part of the interplay of the materials in use (e.g., yarn, size of

Table 3. Transcript of Tracy identifying a stitch unit in her project unit.

1	2	3	4
			
Tracy: "You see, the two parallel lines make a V or a teardrop."	*Tracy: "And you go only under those two."*	*Tracy: "And then yarn over again."*	*Tracy: "Yeah, yarn over."*
Tracy shows Elena how to identify where on the project the next stitch unit goes.	Tracy maneuvers the crochet hook through the stitch unit.	Tracy lets go of the project and Elena takes it.	Tracy turns Elena's project toward her.
Tracy characterizes a stitch unit with a distinct shape and identifies it in Elena's project.	Tracy starts a new stitch unit as if testing whether she correctly identified the prior stitch.	Tracy seems satisfied with her identification of the prior stitch unit.	Tracy continues to help Elena with the next stitch unit.

crochet hook) and the crafter's tension when holding the yarn, all the while performing the same overall routine steps.

To increase the circumference of her circle, Tracy had to identify what a stitch unit looked like and where it was located as well as the location within a stitch where another stitch could be connected to and how (see Table 3). She explained how she identified a stitch unit to one of her neighbors: "You see, the two parallel lines make a V or a teardrop." As Tracy explained she pointed to the place in the project where the stitch unit was in her neighbor Elena's project (see Table 3, panel 1). Tracy identified a stitch unit by looking at the shape the stitch produced when combined with other stitches in a unit. However, rather than looking at a stitch as a flat shape from the top down, her description of "a V or a teardrop" suggests that Tracy identified stitches as 3-dimensional shapes. The "V or (…) teardrop"-like shape is the top of the stitch that is exposed at the rim of the project (see Figure 3(a)). Tracy continued to explain and show in more detail how exactly this unit related to the larger project: "And you go only under those two." To illustrate, she moved the crochet hook through a V shaped stitch unit to create another one (see Table 3, panel 2 to 4). The recognition that stitch units in crochet were three-dimensional made it possible for Tracy to single out interconnected stitch units, to identify their locations, and to point to locations where to attach a new stitch unit. The epistemic engagement with stitch units supported her in continuously producing stitch units and combining them in multiplicative ways.

In a way similar to knitting, Tracy's crochet stitches were multiplied to produce a pattern unit, nested within each row and sector. Once she progressed to the second row, Tracy increased the flat circle's circumference by crocheting two stitch units through each of the stitch units in the prior row (a 2:1 ratio). Typically, to crochet the second row of a flat circle, crocheters build outwards by crocheting two stitches into each stitch on the outer edge of the magic circle, generating a *per-relationship* (i.e., two stitches per sector), suggesting a proportional relationship. The second row of the flat circle, once completed, should include a total of 12 stitches with 2 stitches in each of the six original sectors. The total number of stitches per row and the number of stitches per sector becomes multiplied by 2. Interestingly, to continue to crochet a flat circle, the third row needs to increase by a factor of 3. In practice, this means that the crafter crochets three stitches per sector (instead of two in the previous row).

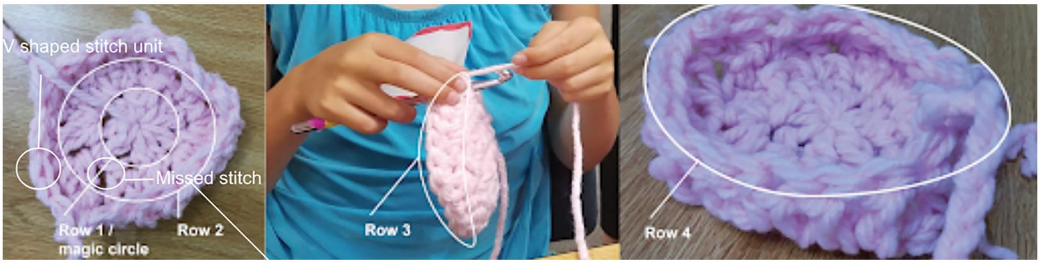


Figure 3. Photograph of Tracy's project highlighting each row.

Though visually the circumference of the circle did indeed increase in Tracy's project, Tracy actually crocheted two stitch units into five of the stitch units of her magic circle (i.e., 2:1 ratio) but only one stitch into the last remaining stitch unit of her magic circle (i.e., a 1:1 ratio; Figure 3(a)). Yet, it was possible for her circle to evenly increase in circumference because she made up for the last missing stitch by producing one stitch unit that was crocheted looser than the prior ones. Although the resulting outcome was not the intended pattern unit (i.e., two stitch units into all six of the stitches of the magic circle), to maintain the evenly increased circumference of the circle toward a flat circular shape, Tracy reformulated her actions to resemble that outcome (i.e., by producing one stitch that was crocheted in a looser way). Had she not done that, the number of stitch units would not have accommodated the even increase in the circumference.

At the project unit level, the proportional relationships from the stitch and pattern units are even more prominent, indicating how the act of crochet involves the execution of three things simultaneously: a) Recognizing stitches as stitch units, b) identifying combinations of stitch units as sectors or pattern units, and c) seeing them in relationship to one another (e.g., seeing sectors inside of sectors). In Tracy's project, the repetition of the stitch unit in connection with other units changed the shape of the stitch unit and of the project unit, showing the interconnectedness of the different layers. The distribution and increase of rectangular stitch units around the flat circle changed the shape of the stitch unit from a rectangle to a trapezoid, where the edge that was connected to the previous stitch was slightly shorter than the opposite edge. Reproducing stitch units in patterned ways also changed the shape of the larger project unit, from a flat circle to a cylinder. To create the walls of her circular bag, Tracy distributed a stable number of stitch units on a circle, rather than increasing the number of stitches per row. To this end, for her third and fourth row (Figure 3(b,c), respectively), Tracy built each stitch unit directly on top of the one below, resulting in walls rising straight up (i.e., 1:1 ratio). Through her implementation of various rates of increase that either maintained the shape of a flat circle to later adjusting it to a cylinder, Tracy arguably developed a sense of multiplicative proportions in crochet as part of her vision of her project unit. This illustrates how different ratios interact uniquely in three-dimensional space and could just as well pull stitch units upward rather than outward, depending on the desired shape of the project unit. Producing and reproducing stitches in patterned ways as part of materialized actions in crochet allowed Tracy to interact with and manipulate the number of stitches per row, and in consequence, the shape of her project unit in three-dimensional space.

In this vignette, materialized action became mathematical engagement as youth got a sense for the number of stitch units per row by 1) identifying a stitch unit and its location in a project unit, 2) distributing an increasing and a stable number of stitches on a circle, 3) shifting between stitch unit, pattern unit (the distribution of stitches per row and sector) and project units, and 4) manually adjusting the tension of stitch units. In the instance of Tracy's crocheted bag, we can see that crochet made it possible for Tracy to explore how different rates of increase materialized in the shape of individual stitch units and the material form of the larger project. In the case of Tracy, learning about the epistemic idea involves shifting between stitch unit, pattern unit (the distribution of stitches per row and sector), and project units, but it also risks inaccuracy, because the crafter can adjust the size of the stitch to make up for an error in the pattern. Yet, it is exactly the need for intentional adjustment in the tension of stitch units that can highlight whether a multiplicative proportional relationship was accurately implemented in crochet.

Margaret's pleated bag: Folding a unit and PR as inches per pleat

For the pleating activity, participants learned to fold pleats with paper and fabric. Unlike knitting and crochet, ratio and proportion manifested within each pleat as inches per pleat, or within each individual unit, which captured the relationship between the hidden layers of fabric that were folded and the top layer that was visible. First, in learning the structure of a pleat, participants folded pleats into paper. This made it possible to practice the routine actions needed to produce a pleat while also making it possible for the material to easily remain in place. Second, participants folded and sewed pleats into fabric and then sewed their pleated fabric into a bag. To support participants in this activity we provided irons and sewing machines. Lastly, participants could create a strap for their bag to complete the projects.

In pleating, the building of a unit and proportional relationship is produced differently compared to knitting and crochet. In place of a stitch unit, in pleating, the folding of fabric becomes the unit (i.e., fold unit). A pleated unit consists of a series of folds, which produce layers of fabric and the three-dimensional shape. To produce a fold unit, pleaters mark the width of the pleat on the fabric, draft score marks at half width of the pleat on both sides of the pleat, and then fold the fabric at four markers. Yet, the soft foldability of the fabric makes personalized unitizing possible. In action, pleaters do not always use measuring tools to pinpoint the locations of each fold and to measure the distance between each fold to produce alignment. Instead, they adjust the size of their pleats by pulling up and flattening down more or less fabric.

One of the youth was Margaret, an 11-year-old, who did not have any prior experience with sewing or pleating. However, over time she constructed a skillfully crafted project that included two evenly shaped pleats she sewed onto another piece of fabric to create a bag. In the process of making her project, she produced six segments by lifting layers of fabric up and folding them flat onto the surface of the fabric, producing three layers of fabric composed of two parts of three segments each. In pleating, proportional reasoning is called for every time one produces a pleat (i.e., inches per pleat). A pleat's proportional relationship is *three is to one as six is to two* (i.e., $3{:}1 = 6{:}2$). The 3:1 ratio is found in the length of the unfolded pleat, which is three times the length of the folded pleat, due to two layers that are folded underneath the top layer. Thus, the pleat reduces the overall length of the fabric by two times the length of the pleat. Another way of identifying the proportional relationship is to consider the amount of fabric needed to produce a pleat of a certain size and identify the per-relationship between them. For example, if a crafter wants to produce a pleat that is one inch wide, they would have to use three inches of fabric (i.e., 3 inches of fabric *per* 1-inch pleat). The calculation for the length of fabric needed for a rectangle with vertical pleats that will be as large as a non-pleated rectangle, can be described as: $L = l + 2np$. Here, L stands for the total length of fabric required, l stands for the length of the fabric in the outcome, n for the number of pleats, and p for the pleat width.

As she worked, Margaret produced fold units by repeatedly pinching fabric up and pressing it down to fold open into the desired design. Margaret's repeated engagement supported an experimentation with a variation of sizes and positions of PR as she produced fold units (see Table 4). She began by planning her project, writing the measurements of each pleat and its position on a piece of paper (see Table 4, panel 1). This practice let Margaret record and store her mathematical engagement and return to it at a later time.

As Margaret turned to the fabric, her plan was to generate a fold unit consisting of three layers, one on top that covered the visible area of the pleat and two folded beneath it. The routine movement of pinching up and pressing down the fabric gave a material form and showed the multiplicative relationship between the layers of fabric that were hidden and the top layer of

Table 4. Transcript of Margaret's changing pinching up and folding down strategies.

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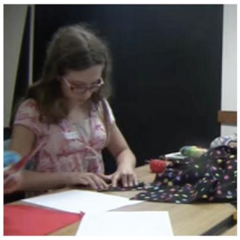
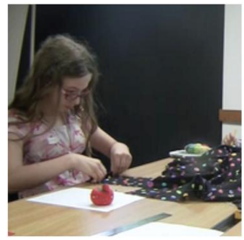
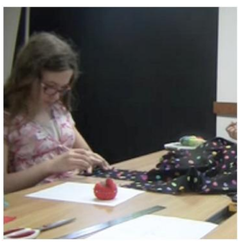
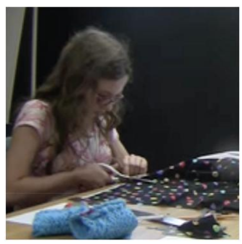
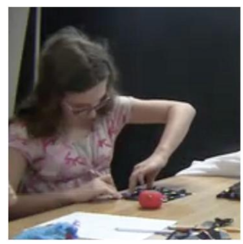
			
		Margaret: "It is hard to pin them in place."	
Margaret writes project measurements on paper.	Margaret pinches her fabric and folds it down. She lets go of the fabric. The fabric folds flat. She repeats this process five times.	Margaret tries to pin the pleat down. She takes the pin off. The fabric flattens.	Margaret repeatedly pinches up her fabric and folds it down before she lets go of the fabric and it folds flat.
She plans her project unit and captures it in place.	She is trying to hold the fabric pleat (i.e., fold unit) in place, similar to the paper measurements (materialized action 1 *folding*).	She explores alternative ways to hold her fold unit in place (materialized action 2 *pinning*).	She continues to try to hold the fold unit in place.
5	**6**	**7**	**8**
			
Margaret: "Ugh."	*Margaret: "It's so hard."*	*Facilitator: "Ok, do you want to take a look at the iron?"*	*Margaret: "I think I need to iron it."*
Margaret breathes strongly and slams her hands on top of her fabric.	Margaret tries to pin the fabric down once more. The fabric folds open again. She slams her hands on top of her fabric again and cuts up her project.	Margaret starts over with a new piece of fabric. The facilitator introduces her to the iron.	After several attempts of pinching up and folding down the fabric, Margaret tries the pinning methods once more. Then she asks for the iron.
It is difficult to maneuver the unfamiliar materials and to fixate a fold unit.	She is taking a break from her problem of fixating the fold unit.	She returns to the fixation problem and introduces the iron.	The iron becomes the third method for fixating the fold unit and the PR (materialized action 3 *ironing*).

fabric that was visible (i.e., 3:1). However, when Margaret lifted her hand and let go of the fabric, the fabric unfolded and quickly reverted into a flat layer without a trace of a fold (see Table 4, panel 2). Material properties (e.g., thickness of the fabric and fabric memory) worked against easily folding, flattening, and creasing the fabric in sustained ways, with the fabric bouncing back into its unfolded state. Thus, it was required to find ways to hold each fold unit in place (see Table 4, panel 3).

Margaret selected where to place the fold units, their size, and the combination of units (i.e., pattern unit). Margaret deliberately chose the position of the pleats in relation to her fabric's

Figure 4. Photograph of Margaret's project.

print pattern, as she produced her own unique fold and pattern units (see Figure 4). She pinched a piece of fabric up, rotated her hands, pressed the fabric down, then let go, watching the pleat unravel. She experimented with her movements, altering the combination of fingers that would hold down the fabric and the pressure applied on the fabric, which also configured her personalized fold unit. Rather than employing exact measurements, Margaret seemingly eyeballed her pleats, and with the routine action of pulling up and pinching down, she demonstrated an understanding of the 3D relationship that was inherent to how PR manifested in fabric. This materialized action of *folding* was different from the measurements that were held in place on paper at the start; the folded PR in fabric was difficult to permanently hold in place (see Table 4, panel 4 and 5). As she let go of the pleated proportion, it unraveled, which called for other materialized actions.

Margaret first attempted to pin the fold units down with the provided pin needles, but this materialized action of *pinning* also did not achieve the expected results because the needles did not flatten the slippery fabric and merely presented support structures for further folding. As Margaret pinned down one side, the other side of the fold unit became unfolded (see Table 4, panel 6). As the conversation at the table turned to ironing, Margaret declared, "I'm gonna use the iron to hold it in place when I pin it. After I iron it, it stays in place for a little time, and then I can pin it." This introduced a third action of *ironing* and was a turning point (see Table 4, panel 7 and 8). Identifying solutions for holding fold units in place allowed Margaret to fix the fold units in place (i.e., creating what's called "fabric shape memory") which held evidence of the multiplicative proportional relationship between fabric layers. Having a reliable strategy for fixing fold units allowed Margaret to move onto pattern units by adding more pleats in combination with each other.

In pleating, the project unit consisted of the folding in fabric, the ironing of the fold and pattern units in place, the sewing of the folds so they would not open again, and then combining their fabric pattern unit with another piece of fabric to make a bag. Where the fold unit and the pattern unit could be replicated in paper, it was the durability and the finished project that could be worn, used to carry other objects, and shown to family members outside of the course that produced another layer of materializing mathematical actions.

In this example, materialized action became mathematical engagement as youth got a sense for folded inches per pleat by testing multiple materialized actions to store fold units in place. In the example of Margaret's pleated bag, we can see that the practice of pinning and ironing expanded the initial proportional practice of pinching-up and pressing-down and showed how epistemic understandings can emerge through materialized actions. Initially, the routine practice of pinching-up and pressing-down increased her fluency with the craft as she repeatedly produced her own fold unit, which fostered an understanding of the overarching proportional relationships between visible and invisible folds. Pinning and ironing, however, contributed a more permanent material representation of the inherent math concepts underlying the production of a fold unit. Through ironing, creases were formed, which stayed intact even after unfolding and helped visualize the sides and vertices within each pleat. The permanence of pleats also held the pleat in place in case the youth let go of the fabric, freeing their hands and minds to move onto another unit type (i.e., moving from fold unit to pattern unit). This made it possible to put fold units in relation and combination with one another to produce pattern units. In turn, at the project unit level, it was possible to produce a complete, functional, and usable project, which has the multiplicative relationships take material form as part of the design. On a fabric-material plain, the ironed pleats were similar to the symbolized demarcations of the measurements that would make up proportions on the piece of paper that Margaret used before starting her fabric PR explorations.

Discussion: Materialized action in mathematical practice

We set out to understand how fiber crafting develops mathematics learning. Through our analysis of knitting, crochet, and pleating across three types of units (stitch/fold unit, pattern unit, and project unit) we can see how learners can engage epistemically in mathematical ideas across different levels and complexities. All three crafts promise ample opportunity to practice proportional relationships: for example, in crochet by partitioning stitch units into sectors and multiplying them in a set pattern. They can zoom in on the unit, creating a unit and deeply understanding a unit's characteristics (i.e., the steps it takes to make a stitch or a fold) as well as the aspects that change a unit (e.g., different tension in knitting, different size of needles). The stitch and fold units are not predetermined, but vary according to the idiosyncrasies of the crafters' tension, yarn type, the needle used, etc. The personalized stitch and fold unit becomes a materialized action that crafters can recognize with both hands and eyes.

Beyond building units, crafters can simultaneously zoom in on the combination of units into pattern units to think about what the combination of units can produce that's larger than the unit itself. For example, crochet makes it possible to bring basic stitch units into relation with one another, producing pattern units (e.g., 6 stitches in row 1, 12 stitches in row two, 18 stitches in row 3) as well as how these patterns need to be distributed in order to produce the kind of project unit they desire.

At the level of the project unit, multiplicative proportional reasoning that is embedded in the performance of the craft comes together most clearly. Especially in knitting and crochet, crafters can engage in epistemic understanding with proportional reasoning by considering just how many stitch and/or pattern units per row or per inch are needed to produce the kind of project they are envisioning. By comparison, the proportional relationship is strongest at the level of the fold unit in pleating. Yet, the pattern and project unit are placed for reinforcing material aspects of the proportional relationship through repetition (pattern unit) as well as fixation (project unit).

Taken together, this advances a notion of materialized action, resituating the "doing of" mathematics as a natural inquiry process that results through emergent patterns between learners and the materialized traces of their actions. To illustrate these micro developmental processes of materialized actions, we designed three fiber crafts activities (crocheting, knitting, and pleating a bag) that supported engagement with proportional reasoning (PR). Each of the craft activities presented unique affordances for PR engagement, which is a persistently challenging area of mathematics (Behr et al., 1992; Boyer & Levine, 2015). The findings showed that PR was produced and performed differently across crafts, providing opportunities for youth to engage with the mathematical concept in a range of ways. All three crafts supported the engagement with personalized units and experiences of different types of units (i.e., stitch and fold units, pattern units, and project units), as well as produced differently shaped units and units that were composed of a different set of steps.

In their own ways, the crafts called for the production, multiplication, and interconnectedness of stitch and fold units that, through repetition (i.e., pattern units), made it possible to create project units. In materialized action, learning about the epistemic idea is moving between types of units and building toward larger constructions. Types of units can be simultaneously and separately engaged. Staying not only at the level of the project unit but simultaneously being aware of more than one unit type highlights deep multi-level engagement with mathematical ideas. Depth of engagement at each level can vary depending on what a person is working on and where in the process of learning a craft one is. By working across units, crafters engage materialized actions that provide opportunities to epistemically engage with proportional reasoning in different ways. Table 5 shows an overview of materialized action as mathematical practice as observed across crafts.

Pattern units evolved because stitch units materialized and sedimented, allowing patterns to emerge instinctively as a consequence. In a constructionist sense, the new object for reflection thus becomes the pattern unit (i.e., the new "object to think with" at this stage of the project) yet it entails the stitch unit and holds space for this thought, highlighting the value of the production of turning a straight line into countable elements for epistemic engagement.

Further, materialized actions integrate (rather than exclude) worldly concreteness, promising another way to relate to math. Units do not have to stay the same within a mathematical activity. Materialized actions recognize the fluidity of units and the production of units within mathematics and propose activities that make it possible to experience and practice this fluidity at a middle school age. This includes introducing worldly qualities (e.g., needle size, tension etc.) to mathematical practice that always underlie mathematical theorization but that all too often can disappear in practice, making math appear more abstract than concrete. Materialized actions as a theoretical idea can guide the design of mathematics learning that is embracing (rather than reducing) the complex concreteness of the world as part of learning key domain ideas, such as proportional reasoning. This in turn holds the promise to engage people with diverse interests in mathematics

Table 5. Overview of materialized action in mathematical practice across crafts.

	Knitted bag	Crocheted bag	Pleated bag
Materialized action in mathematical practice	Getting a sense for the number of stitch units per row by 1) making, unraveling, and remaking stitch units and project units, 2) comparing stitch units of knitting techniques, and 3) comparing project units across paper and yarn.	Getting a sense for the number of stitch units per row by 1) identifying a stitch unit and its location in a project unit, 2) distributing an increasing and a stable number of stitches on a circle, 3) shifting between stitch unit, pattern unit (the distribution of stitches per row and sector) and project units, and 4) manually adjusting the tension of stitch units.	Getting a sense for folded inches per pleat by testing multiple materialized actions to store fold units in place.

learning and unsettle what has previously been conceptualized as a canonical source of mathematics activity, with implications for the design of more equitable learning environments.

The kind of math learning process illustrated in these vignettes cannot be separated from the craft, as it materializes through it. This suggests that we have to broaden our understanding of how the storing of the proportion materializes and relates to epistemic engagement with domain ideas. While Lamon (1996) found that unitizing in the form of viewing units as composites was difficult for youth and developed over time, our work found that partitioning individual actions into composite stitch or fold units happened naturally across the crafts.

Implications and future research

The intervention studied here represents a way to design and integrate math learning environments that are relevant to everyday life contexts, an approach ethno-mathematicians consider highlighting how mathematics are infused across a diverse range of cultural contexts (e.g., Eglash, 2007; Eglash et al., 2006; 2011). It is notable, then, that this study is not intended to be representative of *all* learners and, as the activities were expressly designed to promote the exploration of PR, this study does not set out to take youth perspectives actively into consideration in the design of the activities. The case study approach here was meant to illustrate the possibilities for engaging deeply in mathematical ideas like proportional reasoning through fiber crafts in preparation for future work that seeks to test these designs with larger numbers of youth.

Overall, the findings of this study point toward the opportunities that crafts-based activities open for math engagement and simultaneously suggest future research necessary to support further design iterations toward wider adoption. Future research could include considerations of ways of moving comfortably and fluently across a range of contexts that are explicitly created for domain ideas, such as PR for the mathematics domain. This work offers encouraging ways to think of textile crafts as new tangible manipulatives for engaging in advanced mathematical ideas as well as approaches to designing educational activities to engage learners in persistently challenging areas in mathematics. The promise of this work is that it sets up learners to understand that the ideas of mathematical productions apply in myriad ways in the world around us. Through this activity, youth are producing mathematical works that have a life beyond their workshops, and potentially can act as an artifact to share and remind the learner about some of the big ideas of mathematics.

Materialized action promises opportunities to study further mathematical ideas within crafts and how they are being supported across three levels of units. Where the present study explored PR and considered conditions under which explorations with PR can be brought about through crafts, future work could explore numerous other mathematical concepts through fiber crafts. For instance, nearly every fiber craft involves measuring, and sewing makes use of fraction and decimal operations in measuring (e.g., seam allowances are usually $\frac{3}{5}$ inches). Other possibilities include geometric concepts through quilting, algebra for determining final project length in weaving, and graphing/area under the curve through cross-stitch. One math-adjacent concept present in all three of the crafts explored in this paper, as well as in numerous others, is spatial visualization—moving between 2D and 3D mental representations of space.

Further, performing the crafts with youth was insightful in that it advanced understandings of how similarly and how differently the crafts supported important aspects of proportional reasoning. In the future, designing for engagement with proportional reasoning (or alternative areas of mathematics or STEM fields) supported across a range of tangible fiber crafts could consider the order in which crafts are facilitated to determine any potential benefits of a sequential facilitation. Where the highlighted crafts supported the engagement with PR, other crafts (e.g., weaving and quilting) could become the basis of craft activities related to proportional reasoning, as well. Engaging with the design of additional fiber craft activities toward PR could be beneficial to support a broader range of interests than typically captured in the current K-12 mathematics curriculum.

This work highlighted that teaching how to make units in each craft is important, and that there is a need to streamline this teaching through facilitation strategies, appropriate to the learning audience, including considerations of the initial number of stitches. For example, where our facilitation taught the magic circle with 6 stitches, the magic circle could start with any number of stitches. In future iterations, facilitators can introduce variance and youth's personal exploration of PR as the stitch number of the magic circle sets the basis for the PR that can be explored and multiplied with.

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