Control of Hybrid Dynamical Systems

Ricardo G Sanfelice, Electrical and Computer Engineering Department, University of California, Santa Cruz, CA, United States

Abstract

This entry provides an overview of the control of hybrid dynamical systems. Several frameworks for studying hybrid dynamics are introduced, namely, switched systems, impulsive systems, hybrid automata, and hybrid inclusions, emphasizing key features observed in practical examples. A review of hybrid control approaches is presented, highlighting methodologies that address the complexities inherent to these systems, including uniting control, event-triggered control, synergistic control, and invariance-based control. The entry concludes with insights into the current state of the field and potential research directions.

Notation

- $\mathbb{R}^n$ denotes n-dimensional Euclidean space and $\mathbb{R}$ denotes the real numbers.
- $\mathbb{R}_{\geq 0}$ denotes the nonnegative real numbers, i.e., $\mathbb{R}_{\geq 0}=[0,\infty)$.
- $\mathbb{N}$ denotes the natural numbers including 0, i.e., $\mathbb{N} = \{0,1,\ldots\}$.
- Given $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$, (x, y) is equivalent to $[x^T y^T]^T$.

Key Points

- Hybrid systems integrate continuous and discrete dynamics, effectively modeling complex behaviors encountered in engineering and scientific applications.
- Hybrid inclusion models extend traditional frameworks by incorporating set-valued maps to handle uncertainties, non-deterministic behaviors, non-unique solutions, prematurely ending solutions, and Zeno phenomena.
- Hybrid feedback control techniques address challenges that classical control methods are unable to resolve, offering robust solutions for complex dynamical systems.

Introduction

This entry provides a basic overview to control of hybrid dynamical systems. Several frameworks for studying hybrid dynamics are presented, and control approaches are outlined. These topics are addressed in more depth in the entry entitled *20014. Nonlinear hybrid control systems*.

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By combining continuous and discrete dynamics, hybrid systems are capable of capturing complex dynamical behavior emerging in engineering and science. Hybrid system models can effectively describe the evolution of variables changing continuously as a function of ordinary time and, at certain time instances, change their value discretely upon the occurrence of events or abrupt transitions in the system. Examples of systems exhibiting hybrid dynamics include electric circuits with resettable components, such as relays, switches, and transistors, mechanical systems with impacts — for example, walking robots —, and feedback control systems with multiple modes of operation — for example, autopilots, power systems integrating renewable power, and autonomous vehicles.

Spiking neurons exhibit intricate combination of continuous and discrete dynamics. In fact, most neuron models in the literature have non-smooth and impulsive features. One is typically interested in studying the emergent behavior of interconnections of neurons, in particular, the synchronization of their spiking times, or lack of. Being a natural process, interconnections of neurons are inherently noisy, motivating the study of robustness of their evolution. A single spiking neuron can be modeled by the general N-order, conductance based model given by

$$\begin{bmatrix} \dot{v} \\ \dot{w} \end{bmatrix} = f(v, w) + u^g$$

where $v \in \mathbb{R}$ is the voltage difference across the membrane, w is the $(N-1)$-dimensional vector comprising the gating variables, f is the vector field governing the continuous evolution of (v, w), and u^g is the input stimulus effect. Due to the desired periodic behavior of these quantities, the following (simpler) phase model is typically used to study neuron synchronization:

$$\frac{d\theta}{dt} = \omega + \varphi(\theta)u^\theta$$

The variable θ captures the phase of the neuron with natural frequency $\omega = \frac{2\pi}{T}$, with T being the time between the spiking and reset events of the neuron model above. It changes continuously, as ordinary time evolves. The function φ is called the phase response curve (PRC) characterizing the neurons sensitivity to the given impulsive stimulus u^θ akin of a control input. At time instances where u^θ is nonzero, an instantaneous jump in the evolution of the phase of the neuron occur. The magnitude of the instantaneous phase change depends on the value of the PRC at the current phase. **Fig. 1** shows the evolution of the phase of two interconnected neurons, with phases θ_1 and θ_2, as a function of ordinary time t with a PRC associated with the simplified Hodgkin-Huxley model given by $\varphi(\theta) = -\sin(\theta)$. The synchronization error is captured by the function V given by $V(\theta) = \min\{|\theta_1 - \theta_2|, 2\pi - |\theta_1 - \theta_2|\}$. Unfortunately, there is a distinct lack of systematic methods for analysis of robustness of such interconnections.

Mechanical systems with impacts have trajectories with intervals of continuous evolution and instants where their velocity changes instantaneously. A ball bouncing on the ground is a classic example of a mechanical system with impacts. The ball evolves continuously when off the ground. When an impact occurs, its kinetic energy changes very rapidly, behavior that can be modeled as an instantaneous change. While traveling through the air, the motion of the ball can described by basic kinematics, and when it comes into contact with the surface, the collision can be described by an impact model.

Much like the bouncing ball, a walking robot is a mechanical system with impacts that occur when a foot strikes the ground. During each step, one leg is planted while the other is swinging forward, facing the next impact. Both feet are briefly in contact with the surface during the transition into the next step. A three-link walking robot is shown in **Fig. 2**. The movement of its legs and torso during a step is described by

$$\dot{\theta} = \omega, \qquad D_f(\theta)\dot{\omega} + C_f(\theta, \omega)\omega + G_f(\theta) = Bu$$

where D_f is the inertia matrix, C_f the Coriolis matrix, G_f the gravitation matrix, and B is the actuator relationship matrix. This differential equation captures the continuous evolution of the angles $\theta = (\theta_p, \theta_s, \theta_t)$ and associated angular velocity ω of the walking robot; see **Fig. 2**. The impacts occurring each time a foot makes contact with the ground change the angular velocities instantaneously, according to a restitution law relating the energy before the impact and after it. These can be modeled as discrete dynamics of the form

$$\begin{bmatrix} \theta^+ \\ \omega^+ \end{bmatrix} = G(\theta, \omega)$$

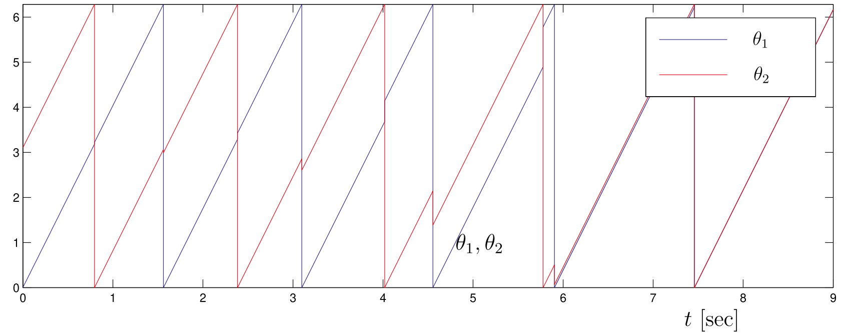
where the map G describes the instantaneous change that occurs upon the impact at the end of each step. At all times, one of the legs is the planted leg while the other is the swing leg, and they switch roles upon each step. This feature is captured by the map G. The resulting model has the variables (θ, ω) evolve continuously in between impacts and discretely at impacts. **Fig. 3** shows trajectories for these variables as a function of ordinary time.

Another class of systems that exhibit hybrid dynamics is feedback control systems with multiple modes of operation. **Fig. 4** describes a feedback system designed to control a physical process using two control algorithms used in different modes: in mode 1, the control algorithm is capable of taking the process variables to nearby a desired setpoint, while in mode 2, a control algorithm assures convergence to such target. A logic variable is used to keep track of the current mode of operation, and, hence, of the controller being employed.

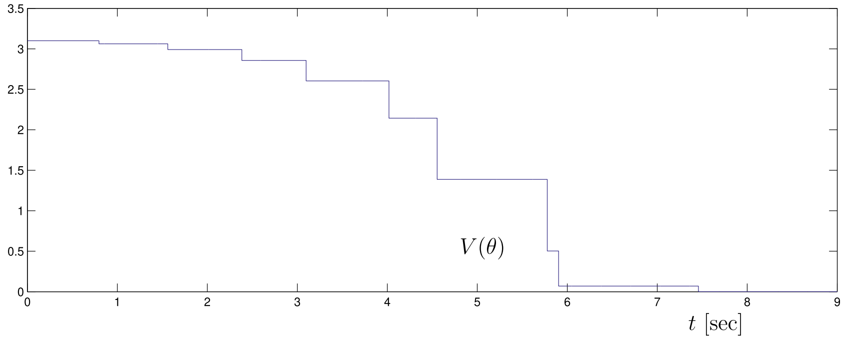
The next section introduces frameworks that are suitable for modeling and analyzing systems exhibiting hybrid dynamics, like the examples above.

Frameworks

This section introduces frameworks for the study of dynamical systems with some of the features seen in the examples in Section "Introduction".



(a) Trajectory (θ_1, θ_2).



(b) Error quantity V along trajectory (θ_1, θ_2).

Fig. 1 A trajectory (θ_1, θ_2) to the simplified Hodgkin-Huxley hybrid model with initial state $(0, 3.1)$ (top) and error quantity V along it, indicating that at around $7.5 sec$, θ_1 and θ_2 are equal (bottom).

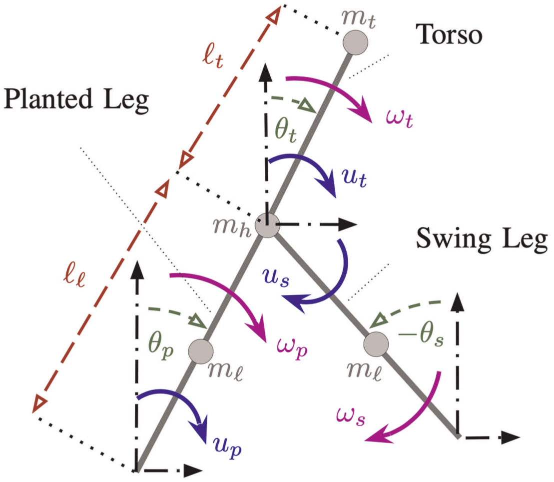


Fig. 2 Walking robot.

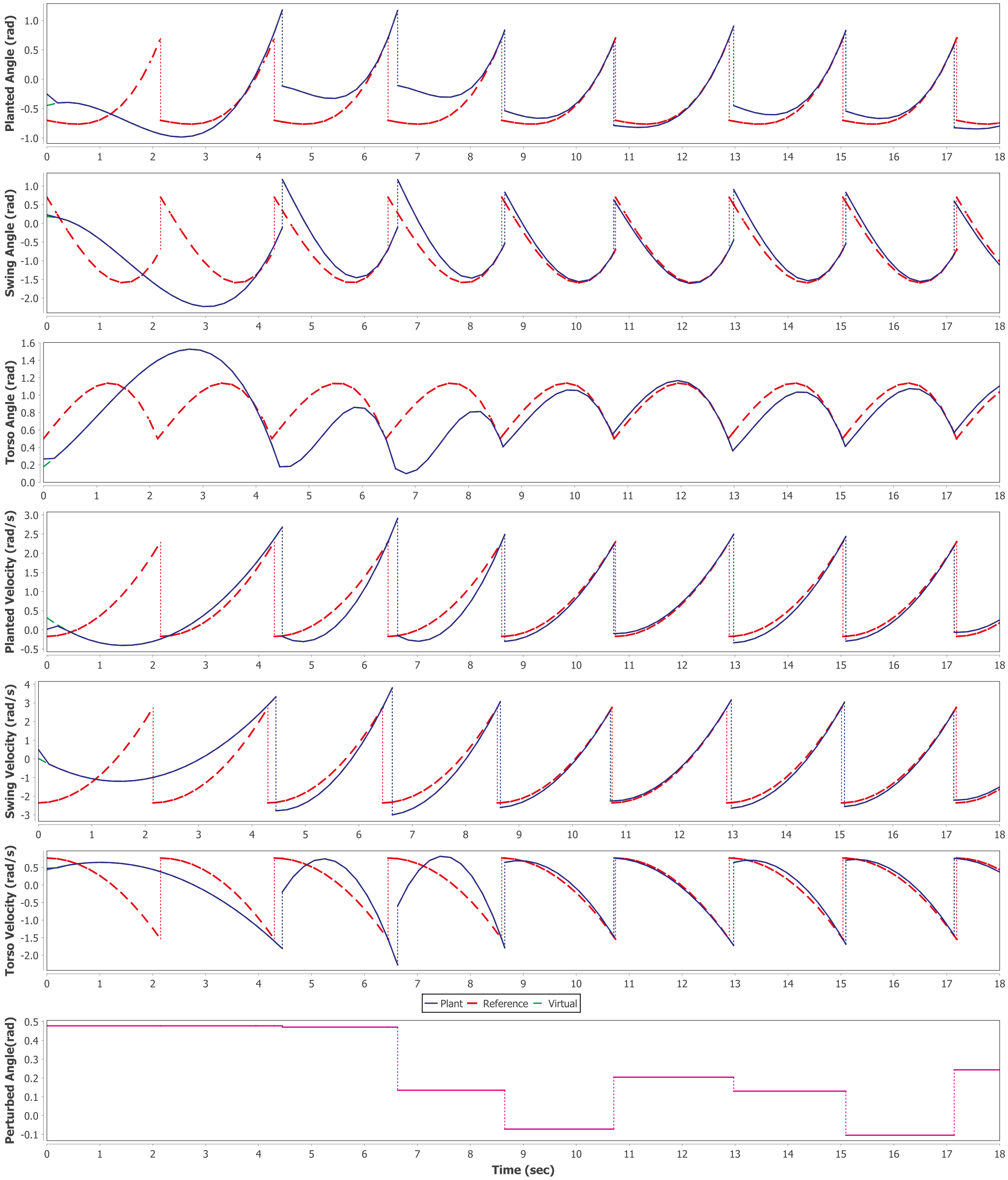


Fig. 3 Simulation results showing limb angles (rad) and velocities (rad/sec), and a perturbation in the step angle. The red dashed line indicates a reference trajectory, the green dashed line indicates a guidance trajectory, and the solid blue line indicates the variables of the walking robot.

Switched Systems

Switched systems are multi-mode continuous-time systems. A switched system consists of a state vector and a family of functions defining its continuous evolution, via a differential equation. The function used at a given time is determined by a switching signal, which changes its value at isolated time instances, called switching times. The state of a switched system does not exhibit instantaneous changes. Switched systems are particularly useful in situations where the switching signal is an external quantity,

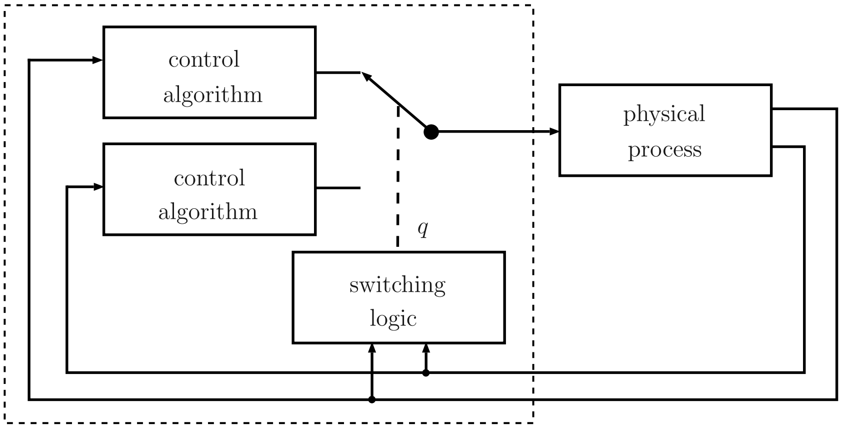


Fig. 4 Multi-mode feedback control.

perhaps unknown (e.g., systems that may exhibit failures), and one is interested in guaranteeing that a desired system behavior is preserved regardless of the value of the switching signal.

Further Reading: (Hespanha and Morse, 1999; Liberzon, 2003)

Impulsive Systems

Impulsive systems are defined by first-order differential equations with impulses at pre-determined times. In between the impulse times, a differential equation governs the evolution of the impulsive system. At each impulse time, the state is instantaneously updated to a new value following an impulsive law via a function that determines the relative change of the state. Impulsive system models emerge in optimal control problems, where the impulses typically corresponds to instantaneous changes in the control signal. These models are also useful when studying systems with state resets for which the impulse times are known in advance, for example, when they are scheduled or known as a function of the initial condition.

Further Reading: (Haddad *et al.*, 2006; Hespanha *et al.*, 2008; Lakshmikantham *et al.*, 1989)

Hybrid Automata

Hybrid automata models allow for multiple modes of operation and instantaneous changes of the state. Similar to switched systems, a hybrid automata includes a logic mode state indicating the operating mode. It is a discrete state that can take values representing modes such as "on" or "off"; "low" or "high"; and "controller 1" or "controller 2." The model also includes a "continuous state" collecting all variables that exhibit changes in between jumps. Similar to impulsive systems, the continuous state can also change at jumps. As a difference to both switched and impulsive systems, the jumps of the hybrid automata are triggered by state conditions that may involve both the discrete and continuous state.

Further Reading: (Lygeros *et al.*, 2003; Nerode and Kohn, 1993; Tavernini, 1987)

Hybrid Inclusions

Hybrid inclusions are mathematical models used to describe systems that exhibit both continuous and discrete behavior. They extend the frameworks outlined above by explicitly incorporating set-valued maps to accommodate uncertainties and non-deterministic behaviors, nonunique solutions, solutions that end prematurely, and Zeno behavior. The evolution of a system modeled by a hybrid inclusion is governed by two types of dynamics:

(I) Flow dynamics: The state x evolves according to $\dot{x} \in F(x, u_c)$ when $(x, u_c) \in C$
(II) Jump dynamics: The state x can instantaneously change to a new state $x^+ \in G(x, u_d)$ when $(x, u_d) \in D$.

A hybrid inclusion is typically described as

$$\mathcal{H} = (C, F, D, G)$$

where

- C is the set of states where the system can evolve continuously, called the flow set;
- F is a set-valued map describing continuous evolution, called the flow map;
- D is the set of states where jumps are enabled, called the jump set;
- G is a set-valued map describing the jumps, called the jump map.

This formulation allows hybrid inclusions to handle uncertainty captured by the set-valued nature of the model and nondeterminism, allowing for the system to evolve along multiple trajectories depending on the flow and jump sets.

The state $x \in \mathbb{R}^n$ captures all of the variables associated to the system. Its time derivative is denoted $\dot{x}$, while x^+ denotes its value after jumps. In several cases, the state x of the hybrid system can contain logic states that take value in discrete sets, as in hybrid automata.

Two parameters are used to specify "time" in solutions to hybrid systems: t, taking values in $\mathbb{R}_{\geq 0}$, and representing the elapsed "real" time; and j, taking values in $\mathbb{N}$, and representing the number of jumps that have occurred. For each solution, the combined parameters (t, j) will be restricted to belong to a hybrid time domain, a particular subset of $\mathbb{R}_{\geq 0} \times \mathbb{N}$. Hybrid time domains corresponding to different solutions may differ.

Hybrid Control Approaches

Uniting Control

The uniting control strategy integrates two feedback controllers — one local and one global — using a logic-based algorithm to determine which controller to apply at any given time. The approach leverages the strengths of both controllers: global controllers can stabilize a set-point across the entire state space but often lack optimal performance in certain regions, while local controllers excel in providing superior performance but only within a limited domain. The uniting control strategy uses a simple hybrid feedback control law, where a logic state governs the selection of the appropriate feedback, ensuring a balanced and effective control across the system.

Further Reading: (Andrieu and Prieur, 2010; Efimov, 2006; Hustig-Schultz and Sanfelice, 2021, 2024; Prieur, 2001; Sanfelice, 2021; Sanfelice and Prieur, 2013; Smith and Sanfelice, 2017)

Event-Triggered Control

Event-triggered control adapts the frequency of control updates to system needs, reacting to predefined events rather than following fixed time intervals in classical periodic discrete-time control. This approach reduces unnecessary computational effort while maintaining system stability and performance, making it suitable for real-time applications in resource-constrained environments. The framework ensures stability by setting thresholds to govern event triggers, striking a balance between responsiveness and efficiency. These events may occur when specific conditions involving state variables, inputs, or outputs are met, prompting instantaneous updates to control variables. For example, sample-and-hold controllers trigger events periodically, updating control inputs at predetermined intervals. In networked control systems, events are tied to data transmission and reception, often occurring asynchronously and governed by network protocols.

Further Reading: (Casau *et al.*, 2021; Chai *et al.*, 2017, 2020; Heemels *et al.*, 2012; Kooi and Sanfelice, 2021; Postoyan *et al.*, 2015, 2019, 2023; Sanfelice, 2021; Tabuada, 2007; Theunisse *et al.*, 2015)

Throw-Catch Control

Throw-and-catch control is a hybrid feedback strategy designed to achieve global asymptotic stabilization of a specific point or set within the state space of nonlinear systems. This approach combines three key components:

- Local Feedback Stabilizers: Control laws effective within specific regions of the state space.
- Open-Loop Schedules: Predefined control signals used to steer trajectories between particular points.
- Bootstrap Feedback Controller: A controller capable of driving trajectories to a region where the local feedback stabilizers and open-loop schedules can take over.

These components are integrated into a hybrid controller, utilizing logic variables and hysteresis-based switching rules to ensure smooth transitions between control modes. The strategy is capable of robustly stabilize a compact set by alternating between these feedback and open-loop control actions. It is particularly effective for nonlinear systems with multiple equilibria.

Further Reading: (O'Flaherty *et al.*, 2008; Sanfelice, 2021; Sanfelice and Teel, 2007)

Synergistic Control

Synergistic control is a hybrid strategy that selects the appropriate state-feedback law based on the values of multiple Lyapunov functions to ensure global asymptotic stabilization of a desired set. The method involves designing a family of pairs consisting of Lyapunov (or Lyapunov-like) functions and corresponding "gradient-like" feedback laws. These pairs are crafted such that each Lyapunov function decreases along system trajectories under its associated feedback law, except at certain problematic points (e.g., local minima or gradient null points), and, at these points, another pair in the family has a Lyapunov function with a smaller value, enabling a switch to its corresponding feedback law. This switching mechanism ensures a consistent decrease in the overall Lyapunov function of the closed-loop system, driving trajectories asymptotically toward the desired set. A family of such pairs satisfying these conditions is termed synergistic.

Further Reading: (Casau *et al.*, 2016, 2019, 2024; Mayhew *et al.*, 2011a,b; Mayhew and Teel, 2011; Sanfelice, 2021)

Supervisory Control

Supervisory control acts as a decision-making layer that oversees and switches between various control algorithms. By continuously monitoring the output of the system, it selects the most appropriate control algorithm to maintain desired performance. The resulting control system features multiple control laws that employ a mechanism acting as a "supervisor" to select the control algorithm to be used. This selection is performed in real time and may involve previous data and decisions made. This hierarchical approach is essential in scenarios like autonomous vehicle navigation or industrial automation, where the system must adapt to changing conditions.

Further Reading: (Goebel *et al.*, 2009; Hespanha *et al.*, 2002; Koutsoukos *et al.*, 2000; Malladi *et al.*, 2016, 2021; Morse, 1996, 1997; Sanfelice, 2021; Sanfelice and Prieur, 2013; Sanfelice *et al.*, 2008; Zucchini *et al.*, 2018)

Passivity-Based Control

Passivity-based control ensures system stability by harnessing the natural energy flow within the system being controlled. It leverages the concept of dissipativity, particularly passivity, to interpret and design feedback control systems through their energy exchanges. Passivity reflects the idea that the energy stored within a system is never greater than the energy supplied to it, with the difference being determined by the initial and final stored energy over time. This stored energy is characterized by a storage function, while the system's power flow is described by the rate of change of the storage function and the product of the system's inputs and outputs. When certain observability conditions are met, passivity provides a framework for control design. By assigning system inputs as functions of outputs, it becomes possible to ensure the rate of change of the system's internal energy is negative, effectively stabilizing the system. This input-output relationship defines the control law, and the method is known as passivity-based control design. The resulting control law manages energy dissipation or conservation, which for hybrid systems, may occur during the continuous or discrete regime, aligning with the system's inherent dynamics. It is particularly effective in applications like robotics and power systems, where energy management is a critical aspect of performance and safety.

Further Reading: (Naldi and Sanfelice, 2011, 2013, 2014; Nanez *et al.*, 2017; Pogromsky *et al.*, 1998; Sanfelice, 2021; Spong *et al.*, 2007)

CLF-Based Control

The design of control algorithms using control Lyapunov functions (CLFs) is a systematic approach for the generation of state-feedback laws for asymptotic stabilization of a set point. A CLF is a scalar function that decreases along the system trajectories for at least one input value, indicating progress toward stabilization of the desired set point. This framework offers clear stability guarantees and is widely applicable in nonlinear and hybrid systems. The primary challenge in this process lies in finding a CLF and ensuring that the resulting feedback law is continuous with respect to the state, which is critical to guarantee robustness. The CLF-based control design approach has been developed for continuous-time and discrete-time systems, and, more recently, for hybrid systems.

Further Reading: (Ames *et al.*, 2012; Goebel *et al.*, 2009; Sanfelice, 2011, 2013a,b, 2016a,b, 2021)

Invariance-Based Control

Invariance-based control focuses on maintaining system states within a predefined set, ensuring that the system trajectories that start in the set remain in the set for all times. These sets are called invariant sets, and when every trajectory from the set stays in the set, the set is said to be forward invariant. In the context of control, the objective of the control algorithm is to restrict the evolution of the system to the desired set, preventing it from reaching unsafe conditions. This approach is integral to applications like collision avoidance and constrained optimization problems. In the context of hybrid systems and control, the evolution of the state of the system is to remain in a given set—a set to be rendered forward invariant—both during flows and at jumps.

Further Reading: (Ames *et al.*, 2017; Chai and Sanfelice, 2019, 2020; Goebel *et al.*, 2008, 2024; Kooi and Sanfelice, 2021; Maghenem and Sanfelice, 2019, 2021; Sanfelice, 2021; Sanfelice *et al.*, 2007; Sanfelice and Teel, 2023; Saoud and Sanfelice, 2021; Wintz and Sanfelice, 2023)

Energy-Based Control

Energy-based control leverages the principles of energy dynamics to synthesize controllers that regulate energy transfer. Similar to passivity-based control, it manages production, storage, and dissipation. Through the combination of flows and jumps, control algorithms with hybrid dynamics are capable of controlling the energy of the system during both continuous evolution and at jumps, leading in particular to instantaneous changes of the energy.

Further Reading: (Astrom and Furuta, 2000; Fantoni *et al.*, 2000; Haddad *et al.*, 2003)

Temporal Logic and Control

Temporal logic translates complex time-dependent requirements into specifications that involve the state and time, such as sequencing tasks or achieving goals within deadlines. Specifications are represented by combinations of functions of the state, called atomic propositions. The combination involves boolean and temporal operators to construct the temporal logic operation. This combination is referred to as a temporal logic formula. Its application in mission-critical domains, like space exploration and automated manufacturing, showcases its versatility.

Further Reading: (Fainekos *et al.*, 2009; Han *et al.*, 2022; Han and Sanfelice, 2020, 2023; Sanfelice, 2021)

Hybrid Model Predictive Control

Hybrid model predictive control (MPC) extends traditional MPC to handle hybrid dynamics. By solving optimization problems that account for mixed dynamics, it predicts future states and computes optimal control actions over a finite hybrid time horizon. This approach is widely used in industries like aerospace, robotics, and advanced manufacturing for its adaptability and performance.

Further Reading: (Altin and Sanfelice, 2019, 2020; Sanfelice, 2018, 2020)

Summary and Outlook

Hybrid control augments classical control theory by incorporating myriad type of state variables and dynamics. Very importantly, it can solve problems that continuous-time and discrete-time control design tools cannot solve. In recent years, the hybrid systems and control has been expanded with powerful control strategies that, inspired from classical control theory, permit systematic analysis and control design of systems where information is available intermittently, the system to control has multiple modes of operation or abrupt changes in its state—such as impacts, faults, and resets—or its dynamics—as in switched or impulsive systems, among many others. Hybrid control theory is a vibrant, fascinating research area, with many opportunities for new theory as well as advancement in emerging applications in science and engineering.

Many of the topics addressed in this entry are revisited in more depth in the entry entitled *20014. Nonlinear hybrid control systems*.

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